LEARNING THROUGH SPORTS™

Mathematics:
Operations with Whole Numbers

The following section of this customized textbook includes material from these skill areas:

Skill Description

2089: add three one-digit numbers
4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

2096: estimate addition solutions
4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

2100: solve addition problems using currency
4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

2101: solve addition problems with decimals
4.MD.2: Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

2103: solve addition problems with fractions
4.NF.3.d: Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4.NF.3.a: Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

2104: <u>solve addition problems with single-digit numbers</u>

4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

2106: <u>solve addition problems with whole numbers</u>

4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

2109: <u>explore concepts of number lines</u>

4.MD.2: Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

2111: <u>use the commutative property of addition</u>

4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

2114: <u>use the zero property of addition</u>

4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

2122: <u>apply division in real-world situations</u>

4.OA.2: Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

2130: <u>solve division problems with whole numbers with and without remainders</u>

4.NBT.6: Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.OA.2: Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

2139: <u>estimate solutions to multiplication problems</u>

4.OA.2: Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

2140: explain inverse relationship of division and multiplication

4.OA.2: Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

2144: solve multiplication problems with whole numbers

4.NBT.5: Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4.OA.2: Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

4.OA.1: Interpret a multiplication equation as a comparison, e.g., interpret 35 is 5 times as many as 7 and 7 times asmany as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

2159: apply subtraction in real-world situations

4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

2170: solve subtraction problems with whole numbers

4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Addition with Carrying (or Renaming)

When addition results in a sum greater than 9 in any place, any amount over 10 is ***carried*** to the next place.

In the ones place of this example, $8 + 8 = 16$.

tens	ones
¹4	8
+	8
5	6

The 6 is written in the **ones** place, and the rest of the amount (1 ten) is ***carried*** over to the **tens** place. That extra ten is added to the other tens in the column. ***Renaming*** and ***regrouping*** are other terms used for this process.

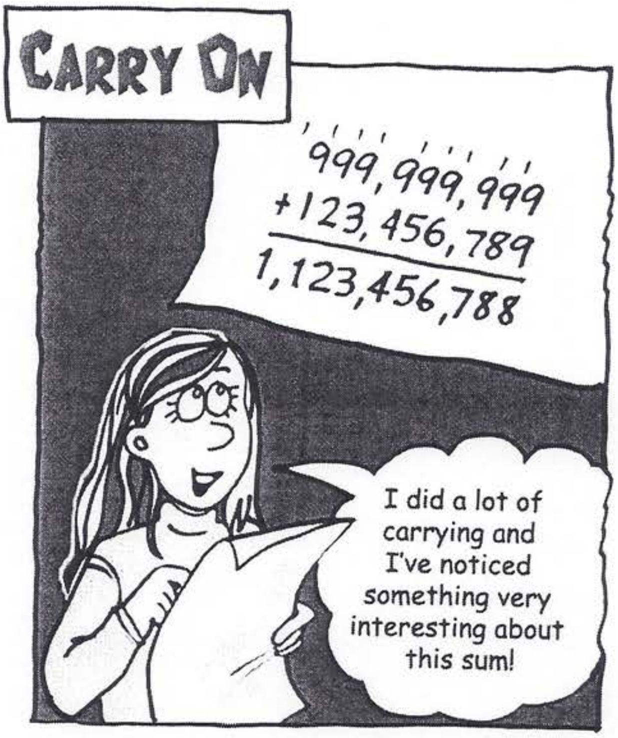
hundreds	tens	ones
¹7	8	5
+1	5	2
9	3	7

In the example, $8 + 5$ equals 13. Since these digits are in the tens place, the sum of these digits has a value of 13 tens.

The amount of 13 tens is ***renamed*** as **3 tens and 1 hundred.** The 3 is written in the **tens** place, and the 1 (value of 100) is added to the other amounts in the **hundreds** column.

Several addends lined up beneath each other form a column. This is called ***column addition.***

$$\begin{array}{r} 17 \\ 88 \\ 42 \\ 90 \\ 5 \\ +\,66 \\ \hline 308 \end{array}$$



Estimating

To **estimate** means to make a reasonable guess. Estimation is a good tool to use when you do not need to end up with a precise count or answer. Rounding is helpful when you estimate.

A school full of kids went to the circus. Each kid dropped 88 peanut shells on the floor. There were 11 kids in each row, and 32 rows in the section. How many peanut shells were dropped on the floor in that section?

Round the 88 peanut shells to 100. Round the 11 kids in a row to 10. Round the 32 rows to 30. Multiply 100 x 10 x 30. The estimated number of shells is 30,000!

One class of 21 students really enjoyed the food at the circus. Each student wanted 2 bags of peanuts, 2 hot dogs, 1 cotton candy, 1 large drink, and 1 ice cream bar. The students had a total of $275.00 between them. Is that enough to pay for their lunch?

Menu

Hamburgers	$3.65
Hot Dogs	$2.75
Fries	$1.75
Ice Cream Bar	$2.10
Peanuts	$ .90
Cotton Candy	$1.85
Popcorn	$2.00
Drinks	
Small	$ .75
Large	$1.25

Round the hot dog price to $3, ($6 for two), the peanuts to $1 ($2 for two), the cotton candy to $2, the drink to $1, and the ice cream bar to $2. This is a total of $13 per person. Round the 21 students to 20. 20 x 13 is $260.00. $275 should be enough.

Money

Decimals are used to write amounts of money.
Money is shown in decimal numbers to the hundredth place.

In a money amount, the decimal point is placed after the dollars (to the right of the one dollars place).

Get Sharp Tip #17
When you do calculations with money, always round off amounts to the nearest hundred (cents).

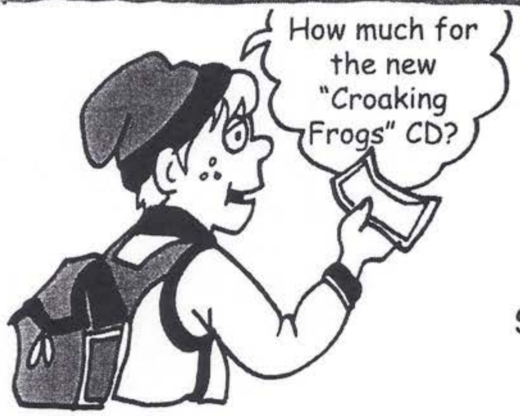
The first place to the right of the decimal is the place of ten cents (one-tenth of a dollar).

The second place to the right of the decimal is the place of one cents (one-hundredth of a dollar).

$765.43

ten cents place *(one tenth of a dollar)*
one cents place *(one hundredth of a dollar)*

The hot, new release from the "Croaking Frogs" sells for eighteen dollars and seventy-four cents.



$99.09 reads ***ninety-nine dollars and nine cents***

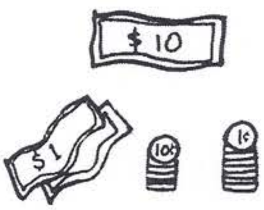
$9.09 reads ***nine dollars and nine cents***

$25.45 reads ***twenty-five dollars and forty-five cents***

$100.30 reads ***one hundred dollars and thirty cents***

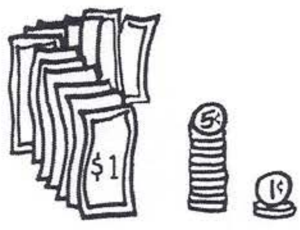
$0.87 reads ***eighty-seven cents***

There are many combinations of coins and bills that make up amounts of money:



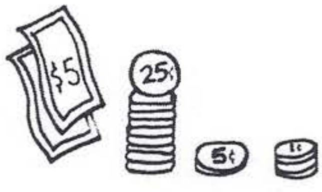
1 ten-dollar bill
+ **2** one-dollar bills
+ **5** dimes
+ **7** pennies

$12.57



12 one-dollar bills
+ **11** nickles
+ **2** pennies

$12.57



2 five-dollar bills
+ **10** quarters
+ **1** nickel
+ **2** pennies

$12.57

Operations with Money

Operations with money are just like operations with decimals, because money amounts are decimals.

Addition

Line up the decimal points carefully in both addends. Align the decimal point in the sum (answer) with the numbers above it.

On the opening weekend of a new superhero movie, kids in our city spent $53,850.50 on movie tickets. Those same kids spent $29,282.35 on candy and popcorn at the movies. How much did they spend all together?

$$\begin{array}{r} \$\ 53,850.50 \\ +\$\ 29,282.35 \\ \hline \$\ 83,132.85 \end{array}$$

Subtraction

Line up the decimal points carefully in both numbers. Then, align the decimal point in the difference (answer) with the numbers above it.

All that popcorn and candy made kids really thirsty. They spent $20,554.25 on drinks at the movie. How much more did they spend on candy and popcorn than on drinks?

$$\begin{array}{r} \$\ 29,282.35 \\ -\$\ 20,554.25 \\ \hline \$\ 8,728.10 \end{array}$$

Multiplication

Multiply as with whole numbers. Tally the total number of places to the right of the decimal point. Count the same number of places from the right in the product.

Anna paid the train fare for herself and four friends when they went into the city to see a movie. The round-trip fare was $5.25 for each rider. How much did Anna spend?

Division

Move the decimal point in the divisor to make it a whole number. Move the decimal point in the dividend the same number of places. Align the decimal point in the quotient with the decimal point in the dividend. Divide as with whole numbers.

Max, a great movie-goer, spent a total of $37.60 on movie tickets last month. On the average, how much did he spend each week?

Operations with Decimals

Adding & Subtracting Decimals

International Falls, Minnesota is the coolest U.S. town—with average temperatures of 36.4°F. Key West, Florida has the warmest average, 77.7°F. What's the difference?

$$\begin{array}{r} 77.7^\circ \\ -\ 36.4^\circ \\ \hline 41.3^\circ \end{array}$$

Step 1: Line up the decimal points of both numbers in the problem.

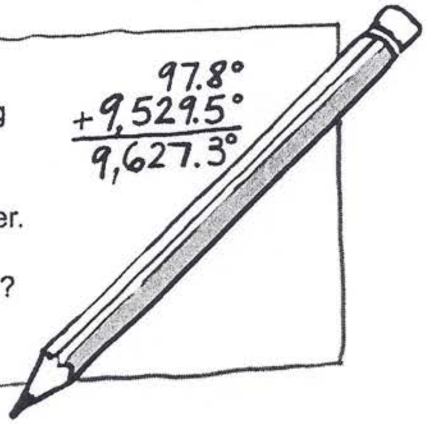
Step 2: Add or subtract just as with whole numbers.

Step 3: Align the decimal point in the sum or difference with decimal points in the numbers above.

The neighbors are complaining about the 97.8° temperatures today. But the temperature on the Sun's surface is 9,529.5° hotter.

What is the Sun's temperature?

$$\begin{array}{r} 97.8^\circ \\ +\ 9{,}529.5^\circ \\ \hline 9{,}627.3^\circ \end{array}$$



The driest city in the U.S. is Yuma, Arizona, with 2.65 inches of precipitation yearly.
The wettest city, Quillayute, Washington, has 39.6 as much.
About how much moisture falls in Quillayute?

$$\begin{array}{r} 2.65 \\ \times\ 39.6 \\ \hline 1590 \\ 2385 \\ +\ 795 \\ \hline 104.940 \end{array}$$

Multiplying Decimals

Step 1: Multiply as you would with whole numbers.
Multiply 2.65 x 39.6 to get 104,940.

Step 2: Count the number of places to the right of the decimal point in both factors (total).
Count the number of places to the right of the decimal point: 2.65 has 2; 39.6 has 1, for a total of 3.

Step 3: Count over from the right end of the product that same number of places.
In the product, count 3 places backward from the right.

Step 4: Insert the decimal point.
Place the decimal point between the 4 and the 9. Quillayute's annual precipitation is about 104.94 inches.

Adding & Subtracting Fractions

How to Add & Subtract Like Fractions

Step 1: If the fractions have like denominators, just add or subtract the numerators. (Denominators stay the same.)

Step 2: Reduce sums or differences to lowest terms.

$$\frac{5}{20}+\frac{3}{20}+\frac{7}{20}=\frac{15}{20}\xrightarrow{\textit{(in lowest terms)}}\frac{3}{4} \qquad \frac{8}{9}-\frac{5}{9}=\frac{3}{9}\xrightarrow[\textit{(in lowest terms)}]{}\frac{1}{3}$$

How to Add & Subtract Unlike Fractions

Step 1: Find the LCM for all denominators and change the fractions to like fractions.

Step 2: Add or subtract the numerators. (Denominators stay the same.)

Step 3: Reduce sums or differences to lowest terms.

$$\frac{1}{3}+\frac{2}{8}=\frac{8}{24}+\frac{6}{24}=\frac{14}{24}\xrightarrow{\textit{(in lowest terms)}}\frac{7}{12}$$

How to Add & Subtract Mixed Numerals

Step 1: Change all mixed numerals to improper fractions.

Step 2: Find the LCM for all the denominators and change the fractions to like fractions.

Step 3: Add or subtract the numerators. (Denominators stay the same.)

Step 4: Reduce sums or differences to lowest terms.

$$7\tfrac{3}{5}-5\tfrac{1}{2}=\frac{38}{5}-\frac{11}{2}=\frac{76}{10}-\frac{55}{10}=\frac{21}{10}=2\tfrac{1}{10}$$

Addition

Addition is the combining of two or more numbers or amounts.

$$52 + 100 + 1{,}000 + 640 = 1{,}792$$

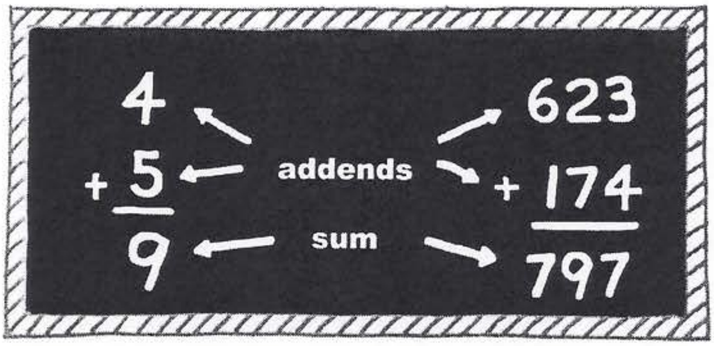
addends *sum*

The symbol for addition is
+

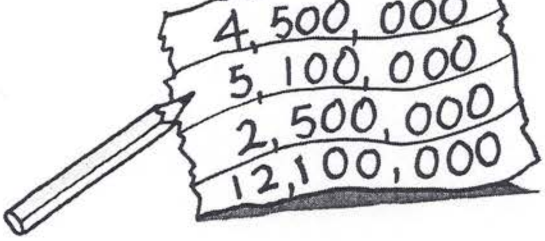
The word used for addition is ***plus***.

The numbers being combined are ***addends***.

The number resulting from addition is a ***sum***.



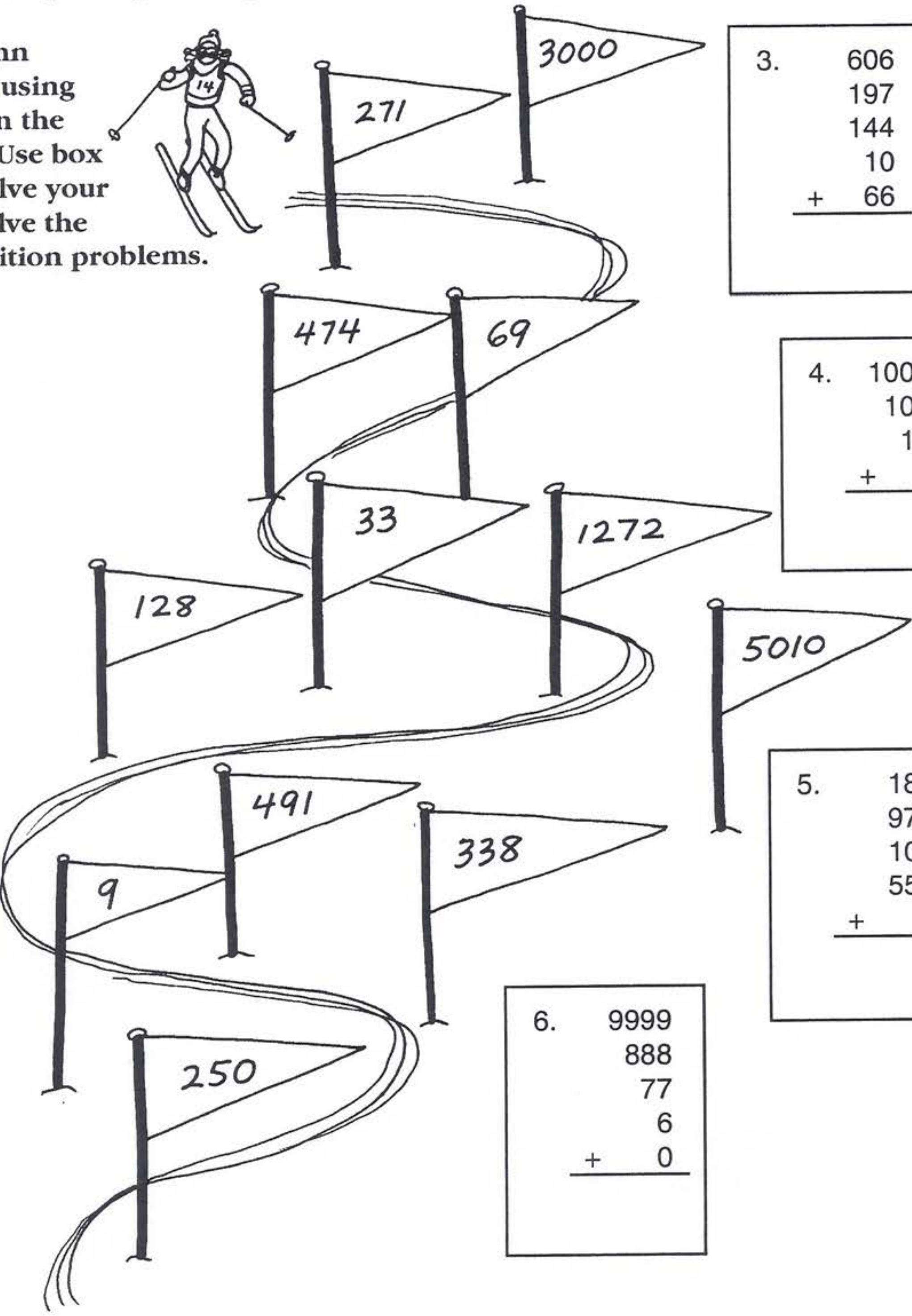
Each year in the U.S., about 5,100,000 people go to hospital emergency rooms with pains in the stomach. About 2,500,000 visit the emergency room with head pain. Another 4,500,000 go with chest pains.



HIGH SPEEDS & TOUGH TURNS

Experts say that the giant slalom takes the most technical skill of any ski event. Skiers race down the mountain over a long, steep, fast course. They must go through a series of gates marked by flags. Spectators also love to watch the downhill slalom, where skiers make high-speed turns to go through the gates at speeds of up to 80 miles per hour!

Write a long column addition problem using all the numbers on the slalom gate flags. Use box #1 to write and solve your problem. Then solve the other column addition problems.



1.

+ ______

2.
```
  5880
   999
  1963
   621
+  511
```

3.
```
   606
   197
   144
    10
+   66
```

4.
```
  10000
   1000
    100
+    11
```

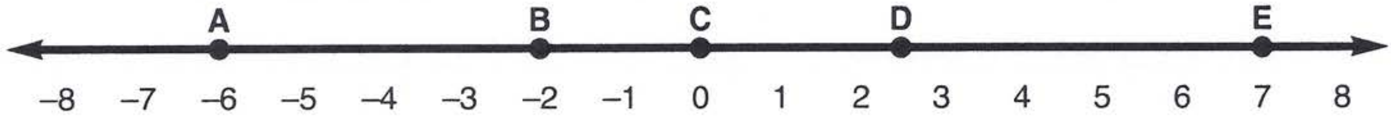
5.
```
   182
   974
   102
   555
+    3
```

6.
```
  9999
   888
    77
     6
+    0
```

Integers

The set of **integers** is the set of numbers including zero and all numbers greater than or less than zero.

Positive integers are greater than zero. **Negative integers** are less than zero.



The number line is a graph of some points corresponding to integers.

The location of each point is called the **coordinate**.

A is the graph of negative 6.
Its coordinate is –6

B is the graph of negative 2.
Its coordinate is –2.

D is the graph of positive 2.5.
Its coordinate is 2.5.

E is the graph of positive 7.
Its coordinate is 7.

Opposites

Every integer has an opposite.
Any two numbers that are the same distance from zero are opposites.

–10 is the opposite of 10

33.5 is the opposite of –33.5

$\frac{1}{2}$ is the opposite of $-\frac{1}{2}$

Absolute Value

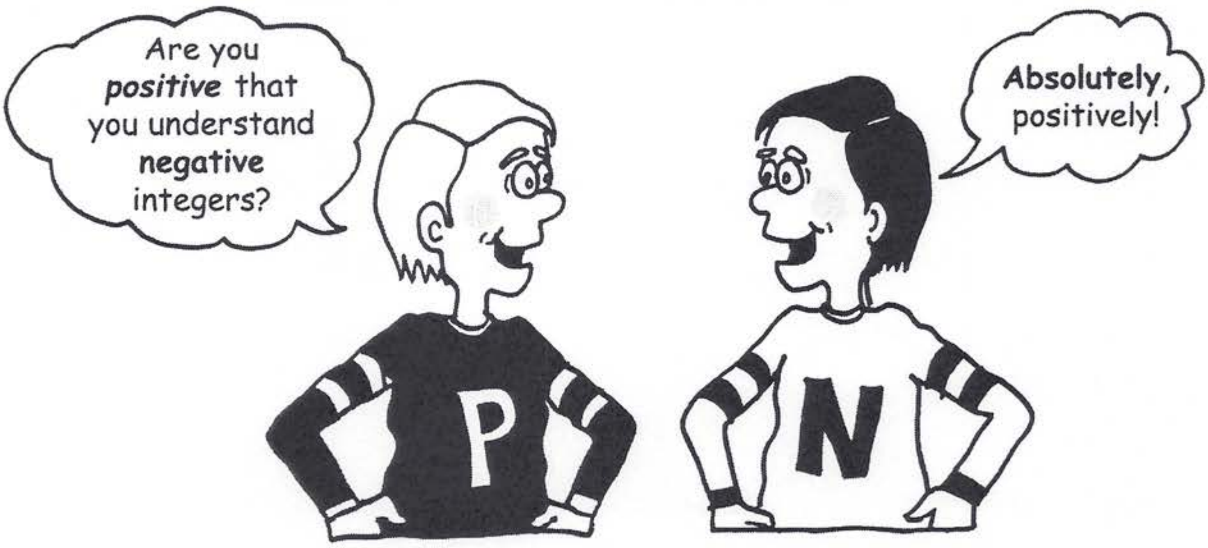
The absolute value of a number is its distance from zero on a number line.

| | is the symbol for absolute value.

| 8 | reads ***the absolute value of 8.***

–25 and 25 have the same absolute value.

| –15 | = 15 **| 15 | = 15**



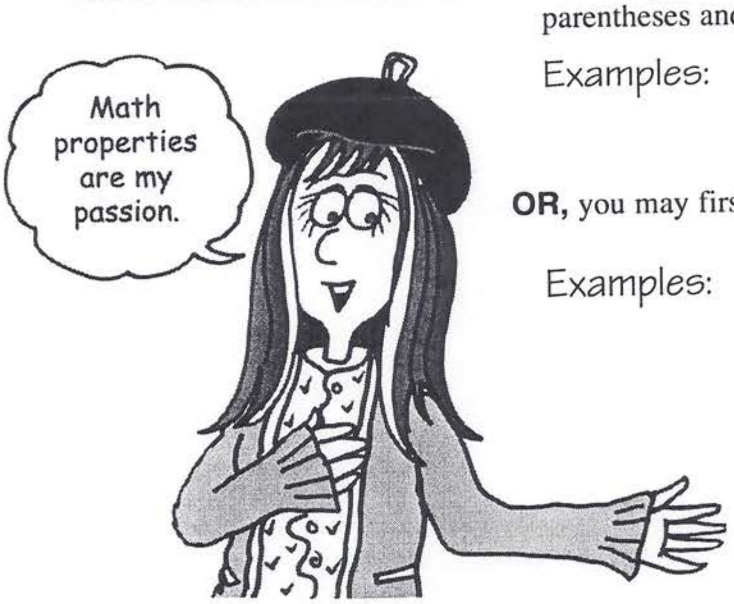
ASK

Dr. Cal Q. Layter

What's so important about properties, anyway?

Answer:
Properties are rules that numbers follow in operations.

- **Knowing the properties will help you understand how numbers work.**
- **Knowing the properties will help you find the correct solutions to many problems.**



Know Your Properties

Commutative Property for Addition:

The order in which numbers are added does not affect the sum.

Examples: $7 + 11 = 11 + 7$

$255 + 144 = 144 + 255$

Commutative Property for Multiplication:

The order in which numbers are multiplied does not affect the product.

Examples: $5 \times 9 = 9 \times 5$

$13 \times 10 = 10 \times 13$

Associative Property for Addition:

The way in which numbers are grouped does not affect the sum.

Examples: $(4 + 3) + 6 = 4 + (3 + 6)$

$(1{,}010 + 584) + 36 = 1{,}010 + (584 + 36)$

Associative Property For Multiplication:

The way in which numbers are grouped does not affect the product.

Examples: $(5 \times 2) \times 4 = 5 \times (2 \times 4)$

$50 \times (30 \times 200) = (50 \times 30) \times 200$

Distributive Property:

To multiply a sum of numbers, you may first add the numbers in parentheses and then multiply the sum.

Examples: $4 \times (6 + 3) = 4 \times (9) = 36$

$9 \times (10 + 5) = 9 \times 15 = 135$

OR, you may first multiply the addends separately, then add the products.

Examples:
$$\begin{aligned} 4 \times (6 + 3) &= (4 \times 6) + (4 \times 3) \\ &= 24 + 12 \\ &= 36 \end{aligned}$$

$$\begin{aligned} 9 \times (10 + 5) &= (9 \times 10) + (9 \times 5) \\ &= 90 + 45 \\ &= 135 \end{aligned}$$

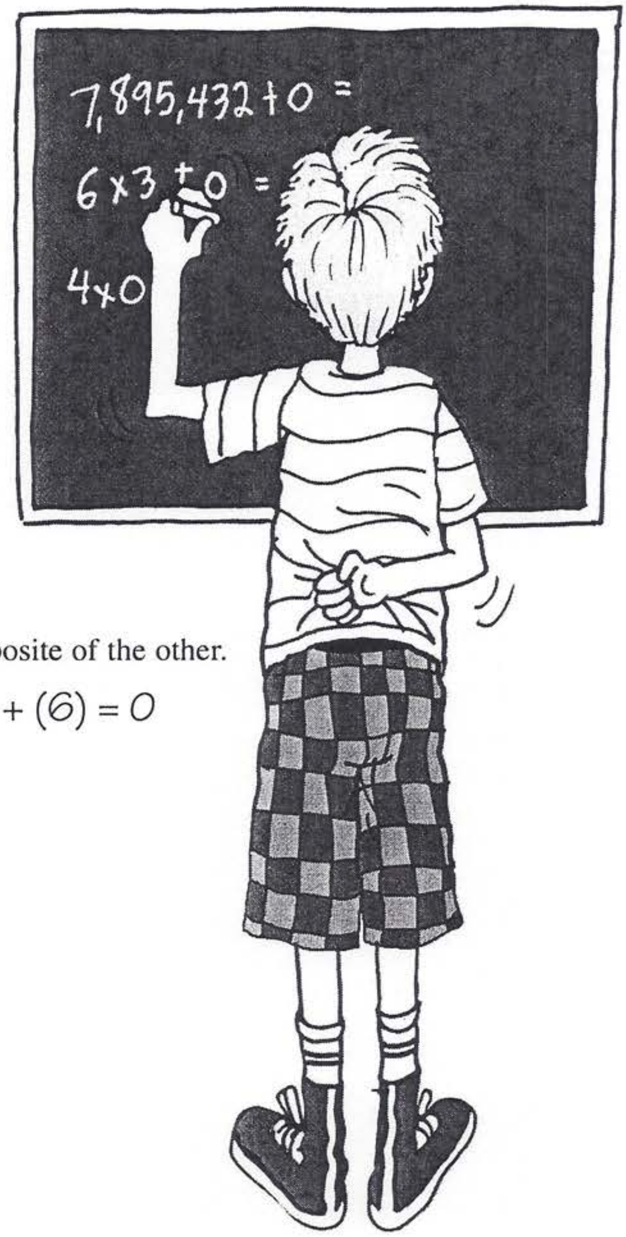
Identity Property For Addition:

The sum of zero and any number is that number.

Examples: $9 + 0 = 9$

$486 + 0 = 486$

$0 + 68{,}117 = 68{,}117$



Identity Property For Multiplication:

The product of 1 and any number is that number.

Examples: $6 \times 1 = 6$

$1 \times 25 = 25$

$7{,}993 \times 1 = 7{,}993$

Opposites Property:

If the sum of two numbers is 0, then each number is the opposite of the other.

Examples: -6 is the opposite of 6 because $-6 + (6) = 0$

-42 is the opposite of 42 because $-42 + (42) = 0$

Zero Property for Addition:

The sum of zero and any number is that number.

Examples: $0 + 8 = 8$

$407 + 0 = 407$

Zero Property for Multiplication:

The product of zero and any number is zero.

Examples: $0 \times 8 = 0$

$6 \times 0 = 0$

$5{,}000 \times 0 = 0$

Equation Properties:

When adding or subtracting the same number or multiplying or dividing by the same number on both sides of an equation, the result is still an equation.

Examples:

$n - 8 = 7,$	$5n = 30$
$n - 8\,(+8) = 7\,(+8)$	$5n\,(\div 5) = 30\,(\div 5)$
$n = 15$	$n = 6$

Finding the Missing Operations in Word Problems

- Watch for these words in problems or directions. They are signals for ***addition***!

sum	*total*
together	*both*
all together	*increased by*
average	

- Watch for these words in problems or directions. They are signals for ***subtraction***!

difference	*less than*
left over	*remain*
take away	*have left*
fewer than	*much more*
much less	*reduced by*

- Watch for these words in problems or directions. They are signals for ***multiplication***!

times	*a product of*
how many times	*twice as much as*

- Watch for these words in problems or directions They are signals for ***division***!

divided by	*sharing*
average	*equal parts*
half as much	*any fraction*

In the past five years, 57 injuries sent members of the Tioga Tumblers gymnastics team to the emergency room. This year, there have been 9 injuries requiring emergency room visits. What is the average number per year over the six years?

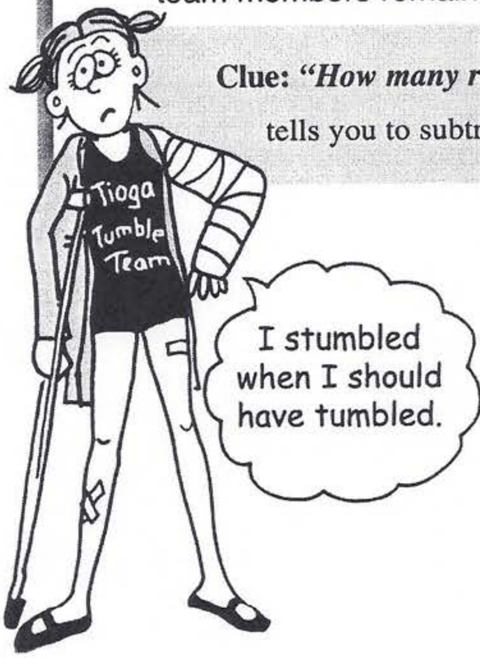
Clue: *"What is the average"* tells you to add and divide.

Roxy had four times as many injuries as Maxie, who had 7. How many injuries did Roxy have?

Clue: *"four times as many"* tells you to multiply.

The team has 38 members. This year, 19 have suffered injuries. How many team members remain uninjured?

Clue: *"How many remain"* tells you to subtract.



Division

Division is repeated subtraction. When you divide, you are subtracting the same number over and over again.

You can subtract 8 from 48 six times: $48 - 8 - 8 - 8 - 8 - 8 - 8 = 0$

Or, you can divide $48 \div 8$ and get 6.

$48 \div 8 = 6$ means there are 6 groups of 8 in 48.

The symbol for division is $\overline{)}$ or $\div$.

Division is a way of finding out how many times one number (the **divisor**) will fit into another number (the **dividend**).

The number resulting from division is the **quotient**.

If a divisor does not fit an even number of times into a dividend, there will be a number left over. This is called the **remainder.**

$$\begin{array}{r} 8 \leftarrow \textit{quotient} \\ \textit{divisor} \rightarrow 9\overline{)72} \leftarrow \textit{dividend} \end{array}$$

$$\underset{\textit{dividend}}{\underset{\uparrow}{9{,}633}} \div \underset{\textit{divisor}}{\underset{\uparrow}{3}} = \underset{\textit{quotient}}{\underset{\uparrow}{3{,}211}}$$

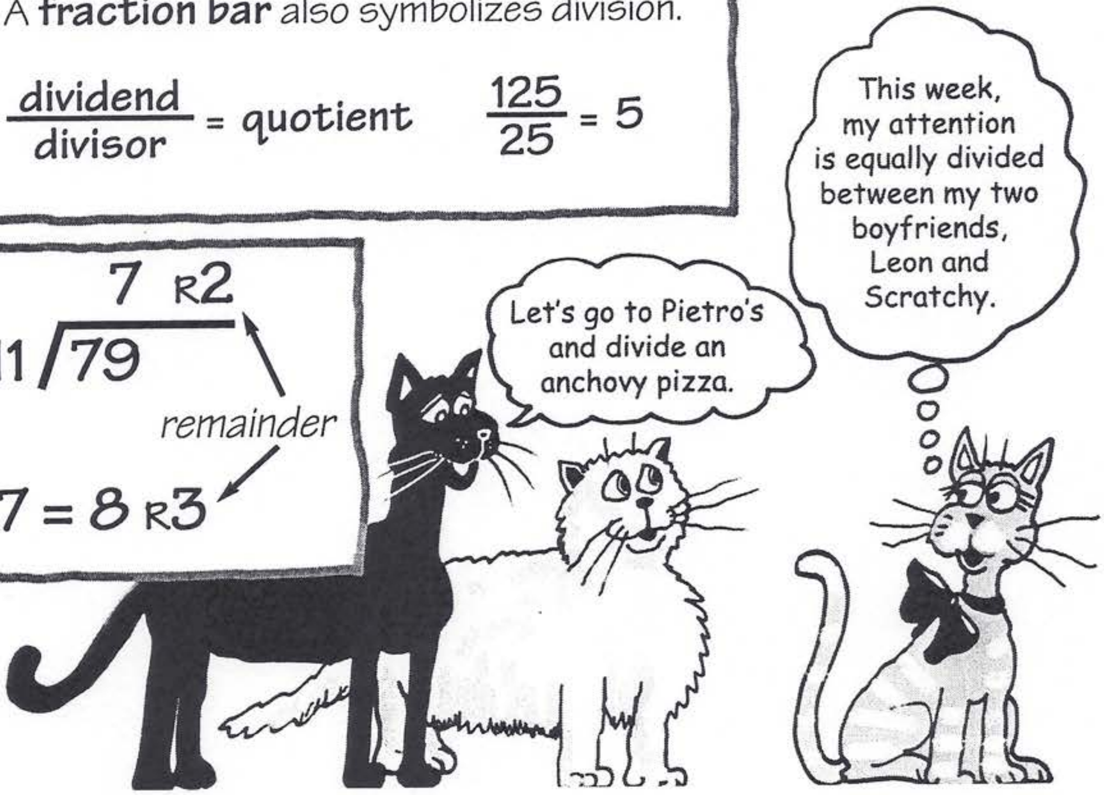
A **fraction bar** also symbolizes division.

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient} \qquad \frac{125}{25} = 5$$

$$\begin{array}{r} 7 \text{ R}2 \\ 11\overline{)79} \end{array}$$

remainder

$$59 \div 7 = 8 \text{ R}3$$



Division with One-Digit Divisors

$$
\begin{array}{r}
159 \text{ R } 2 \\
5\,\overline{)\,797} \\
-5 \\
\hline
29 \\
-25 \\
\hline
47 \\
-45 \\
\hline
2
\end{array}
$$

Step 1: Does 5 go into 7? (yes—1 time)

Write the 1 above the 7.
Multiply 1 x 5. Write the product under the 7.
Subtract 7 – 5 (= 2).
Bring the next digit (9) down next to the 2.

Step 2: Does 5 go into 29? (yes—5 times)

Write the 5 above the 9.
Multiply 5 x 5. Write the product under 29.
Subtract 29–25 (= 4).
Bring the next digit (7) down next to the 4.

Step 3: Does 5 go into 47? (yes—9 times)

Write the 9 above the 7.
Multiply 9 x 5. Write the product under 47.
Subtract 47 – 45 (= 2).
Write the remainder (2) next to the quotient.

Division with Multiples of Ten

$$36{,}000 \div 100 =$$

Step 1:
Place a decimal point after the dividend:
36,000.

Step 2:
Move the decimal point one place to the left for each zero in the divisor:
360.00
(The decimal point was moved 2 places because 100 has 2 zeros.)

Step 3:
Drop any zeros to the right of the decimal point:
$36{,}000 \div 100 = 360$

Division with Larger Divisors

When the divisor has more than one digit, division problems can get very tricky.

Here are some steps to help you handle this process without feeling baffled.

```
      370 R 8
32 ) 11,848
    -96
     224
    -224
      08
     - 0
       8
```

Step 1: Does 32 go into 1? (no)

Step 2: Does 32 go into 11? (no)

Step 3: Does 32 go into 118? (yes)

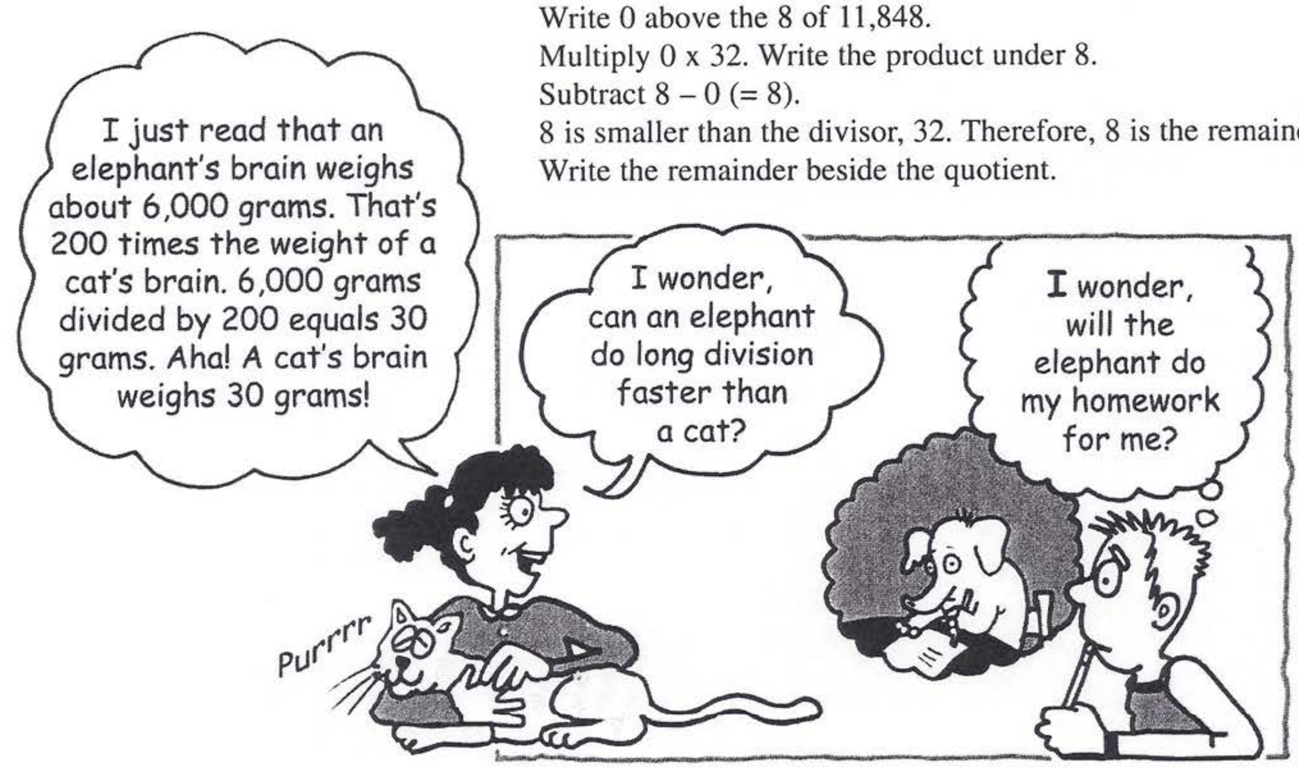
Round 32 to the closest 10. (30)
Estimate the number of 30s in 118. *(about 3)*
Write 3 above the 8 of 118.
Multiply 3 x 32. Write the product under 118.
Subtract 118 – 96 (= 22).
Bring down the next digit (4) beside the 22.

Step 4: Does 32 go into 224? (yes)

Round 32 to 30 again.
Estimate the number of 30s in 224. *(about 7)*
Write 7 above the 4 of 1,184.
Multiply 7 x 32. Write the product under the 224.
Subtract 224 – 224 (= 0).
Bring down the next digit (8) beside the 0.

Step 5: Does 34 go into 8? (no)

Write 0 above the 8 of 11,848.
Multiply 0 x 32. Write the product under 8.
Subtract 8 – 0 (= 8).
8 is smaller than the divisor, 32. Therefore, 8 is the remainder.
Write the remainder beside the quotient.



Estimate

An **estimate** is an approximate solution to a problem. Sometimes, a problem does not need an exact answer, and an estimation is a quick or practical solution.

Get Sharp Tip #33

Don't forget to use rounding when you estimate. It's a great estimation tool.

The Problem:

The 34-member bowling team spends plenty of money while they practice for their tournaments. They have 13 practices a week during their 8-week training camp. Each practice session costs $9 per team member.

The bowlers and their families raised $25,000 at fundraising events.

Will that be enough to pay for the practice sessions?

Round the 34 players to 30.

Round the 13 practices to 10.

Round the 8 weeks of the camp to 10.

Multiply 30 x 10 x 10 x 10 to get $30,000.

Extend a Pattern

Game	Louie's Scores
1	156
2	162
3	168
4	174
5	180
6	186
7	192
8	
9	
10	

Sometimes the best strategy is to look for a pattern in the data. Then extend (continue) the pattern to find a solution to the problem.

The Problem:

The table shows Louie's scores in the first 7 games of the bowling tournament.

What do you predict will be his score in the next three games?

Notice the pattern: Louie's score increased by 6 points from each game to the next.

Extend the pattern to find his score in games 8, 9, and 10. (He will score 198, 204, and 210!)

Dividing a Number by Itself

ANY number divided by itself yields **1**!

$$65 \div 65 = 1$$

No matter HOW BIG the number is, the quotient is still ONE.

$$999{,}999{,}999 \div 999{,}999{,}999 = 1$$

Dividing a Number by One

ANY number divided by 1 yields that number.

$$95 \div 1 = 95$$

$$700{,}000 \div 1 = 700{,}000$$

Divisibility

A number is ***divisible*** by another number if the quotient of the two numbers is a whole number. (50 is divisible by 5 because the quotient is a whole number, 10.)

A number is ***divisible by 2*** if the last digit is 0, 2, 4, 6, or 8.

A number is ***divisible by 3*** if the sum of its digits is divisible by 3.

A number is ***divisible by 4*** if the last two digits are divisible by 4.

A number is ***divisible by 5*** if the last digit is 0 or 5.

A number is ***divisible by 6*** if the number is divisible by both 2 and 3.

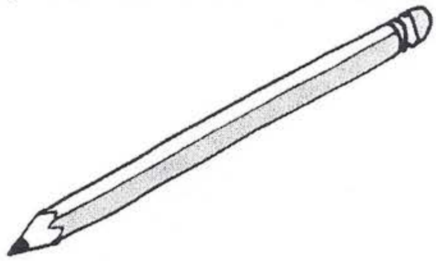
A number is ***divisible by 8*** if the last three digits are divisible by 8.

A number is ***divisible by 9*** if the sum of its digits is divisible by 9.

A number is ***divisible by 10*** if the last digit is 0.

Get Sharp Tip #10

There is **no** division by zero. It's impossible.



MULTIPLICATION AND DIVISION ARE RELATIVES

Multiplication and division are **opposite** (inverse) operations.

$3 \times 7 = 21$

$21 \div 3 = 7$

and

$21 \div 7 = 3$

A multiplication problem can be checked with division.

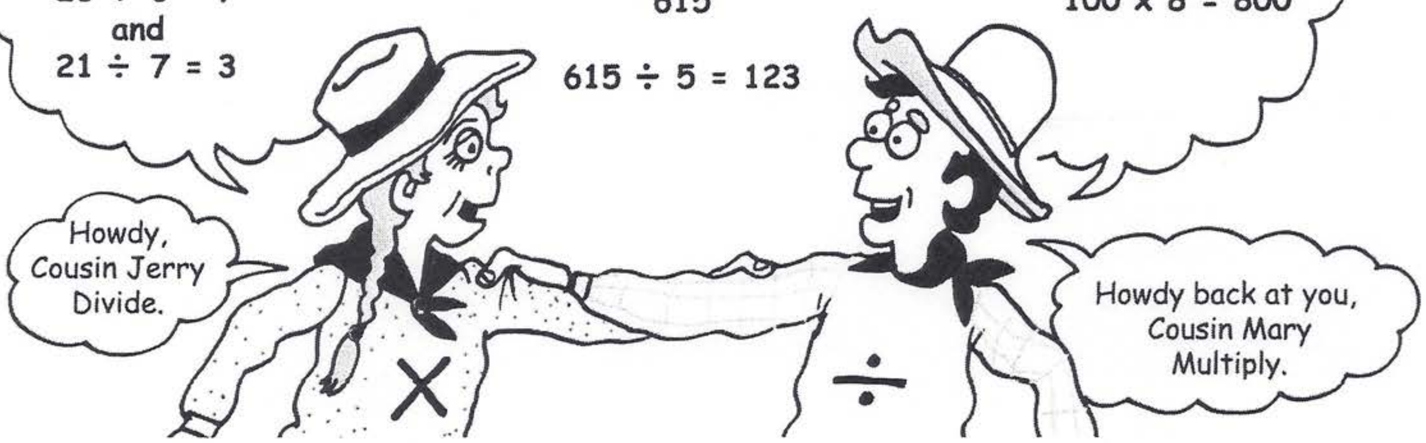
$$\begin{array}{r} 123 \\ \times\ 5 \\ \hline 615 \end{array}$$

$615 \div 5 = 123$

A division problem can be checked with multiplication.

$800 \div 8 = 100$

$100 \times 8 = 800$



Multiplication

Multiplication is repeated addition. When you multiply, you are adding the same number over and over again.

The symbol for multiplication is

× or • .

The word used for multiplication is **times.**

The numbers being multiplied are **factors.**

The number resulting from multiplication is a **product.**

You can add 6 + 6 + 6 + 6 + 6 + 6 + 6 to get 42.

Or, you can multiply 6 x 7 and get 42.

6 x 7 means seven groups of six.

$$\begin{array}{r} 6 \\ \underline{\times\ 7} \\ 42 \end{array} \qquad \leftarrow \text{factors} \rightarrow \qquad \begin{array}{r} 111 \\ \underline{\times\ \ 4} \\ 444 \end{array}$$

$$42 \leftarrow \text{product} \rightarrow 444$$

$$\underset{\text{factor}}{3{,}333} \times \underset{\text{factor}}{3} = \underset{\text{product}}{9{,}999}$$

Multiplying by One

ANY number multiplied by one has a product the same as the number!

$65 \times 1 = 65$

$999{,}999 \times 1 = 999{,}999$

Multiplying by Zero

ANY number multiplied by zero is **0**!

$65 \times 0 = 0$

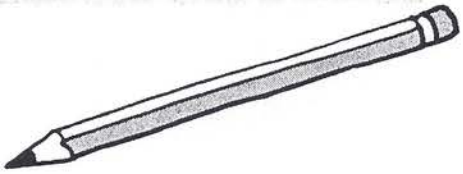
No matter HOW BIG the number is, the product is still ZERO.

$0 \times 999{,}999{,}999{,}999 = 0$

Multiplication with Renaming

Sometimes you will need to **rename** (or regroup) numbers to complete a multiplication task. Here's how it works.

	thousands	*hundreds*	*tens*	*ones*
		2	5	
		8	3	9
x				6
	5,	0	3	4



Step 1: Multiply the ones. 6 x 9 = 54 ones.
Rename the 54 ones as 5 tens and 4 ones.
Write the 5 above the tens column, and the 4 in the ones place in the product.

Step 2: Multiply the tens: 6 x 3 = 18.
Add the 5 tens. 18 + 5 = 23 tens.
Rename the 23 tens as 2 hundreds and 3 tens.
Write the 2 above the hundreds column, and the 3 in the tens place in the product.

Step 3: Multiply the hundreds: 6 x 8 = 48 hundreds.
Add the 2 hundreds: 48 + 2 = 50 hundreds
Rename the 50 hundreds as 5 thousands and 0 hundreds.
Write the 0 in the hundreds place, and the 5 in the thousands place in the product.

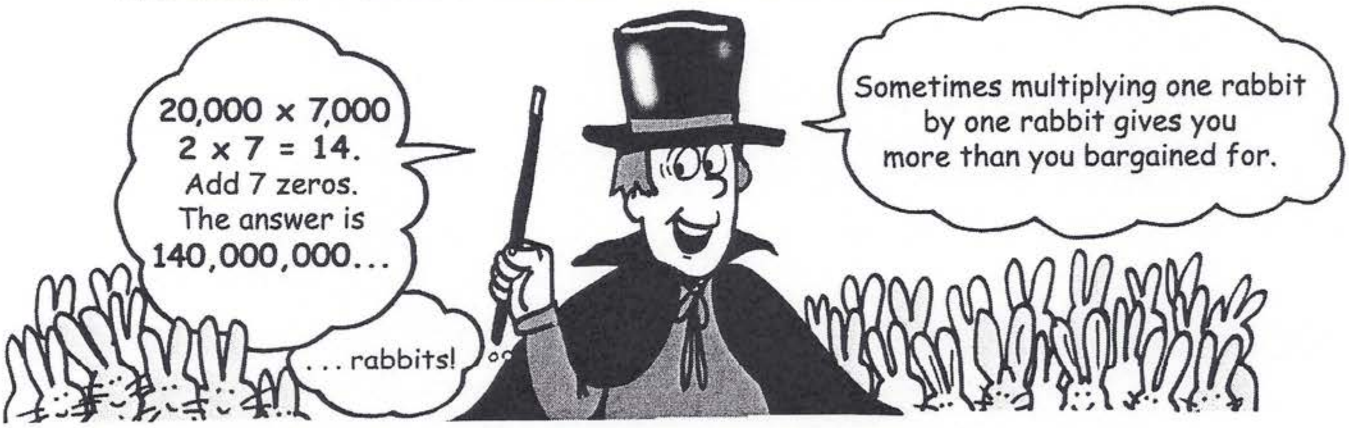
Multiplication with Multiples of Ten

30 x 800 =

Step 1:
Drop the zeros and rewrite the problem as a basic multiplication fact.
3 x 8

Step 2:
Find the product.
3 x 8 = 24

Step 3:
At the end of the product, write the same number of zeros you dropped.
24,000



Multiplication by Larger Numbers

ten thousands	thousands	hundreds	tens	ones
		3	6	8
	x	2	5	7
	2	5	7	6
1	8	4	0	
+7	3	6		
9	4,	5	7	6

Get Sharp Tip #9

There are 100 basic multiplication facts with factors 1-10. If you learn the first 55, you will know all 100—because the order of the factors does not change the product.

Step 1: Multiply by ones.

Multiply 7 x 8 (7 x 8 = 56 ones)
Rename the 56 ones as 5 tens and 6 ones.

Multiply 7 x 6 (7 x 6 = 42 tens).
Add the 5 tens. (42 + 5 = 47 tens)
Rename the 47 tens as 4 hundreds and 7 tens.

Multiply 7 x 3 (7 x 3 = 21 hundreds).
Add the 4 hundreds. (21 + 4 = 25 hundreds)
Rename the 25 hundreds as 2 thousands and 5 hundreds.

Step 2: Multiply by tens.

Multiply 5 x 8 (5 x 8 = 40 tens)
Rename the 40 tens as 4 hundreds and 0 tens.

Multiply 5 x 6 (5 x 6 = 30 hundreds).
Add the 4 hundreds. (30 + 4 = 34 hundreds)
Rename the 34 hundreds as 3 thousands and 4 hundreds.

Multiply 5 x 3 (5 x 3 = 15 thousands).
Add the 3 thousands (15 + 3 = 18 thousands).
Rename the 18 thousands as 1 ten thousand and 8 thousands.

Step 3: Multiply by hundreds.

Multiply 2 x 8 (2 x 8 = 16 hundreds)
Rename the 16 hundreds as 1 thousand and 6 hundreds.

Multiply 2 x 6 (2 x 6 = 12 thousands).
Add the 1 thousand. (12 + 1 = 13 thousand)
Rename the 13 thousands as 1 ten thousand and 3 thousands.

Multiply 2 x 3 (2 x 3 = 6 ten thousands).
Add the 1 ten thousand. (6 + 1 = 7 ten thousands.)

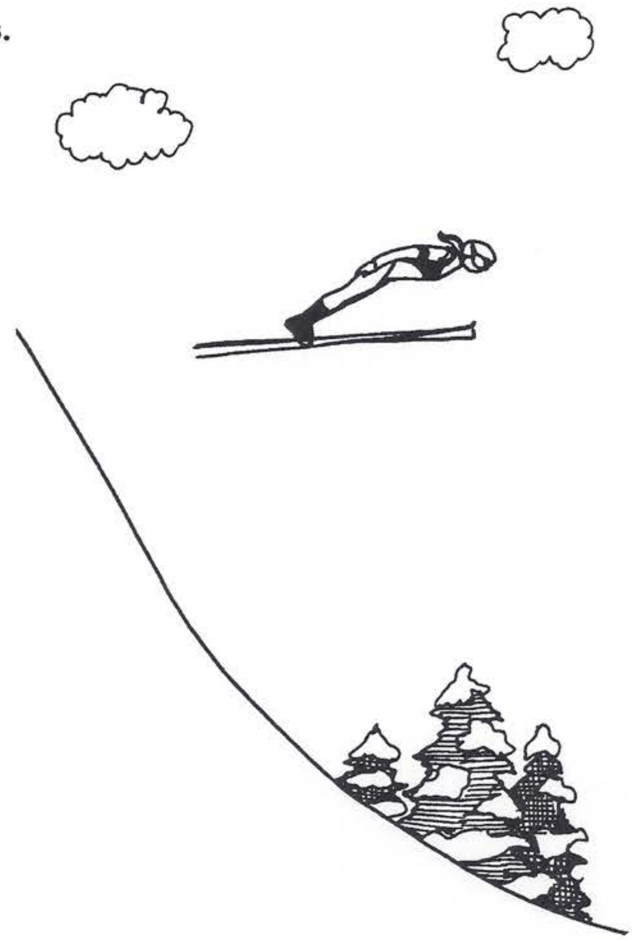
Step 4: Add the columns.

Multiply & Divide with Multiples of 10

SAILING WITHOUT A SAIL

It's spectacular . . . breathtaking . . . awesome! Crowds at the Winter Olympics always love to watch the ski jumpers sailing through the air. Spectators hold their breath until the skier lands safely on the ground! Skiers gain points for strong take-offs, smooth flights, clean landings, and distance. Skiers take off into the air from jumps as high as 120 meters and sail for hundreds of feet.

Use multiplication to figure out these distances.



1. 23 meters x 10 = ____________
2. 23 meters x 100 = ____________
3. 31 meters x 30 = ____________
4. 111 meters x 1,000 = ____________
5. 505 meters x 10 = ____________
6. 2,222 meters x 400 = ____________
7. 717 meters x 10,000 = ____________
8. 4,024 meters x 20 = ____________
9. 70 meters x 40 = ____________
10. 250 meters x 1,000 = ____________

Use division to figure out these distances.

11. 4,400 meters ÷ 10 = ____________
12. 4,400 meters ÷ 100 = ____________
13. 4,400 meters ÷ 200 = ____________
14. 1,000 meters ÷ 10 = ____________
15. 1,000 meters ÷ 100 = ____________
16. 330 meters ÷ 10 = ____________
17. 880,000 meters ÷ 1,000 = ____________
18. 700 meters ÷ 70 = ____________
19. 5,600 meters ÷ 80 = ____________
20. 61,070 meters ÷ 10 = ____________

Olympic Fact

Judges stand at one-meter intervals along the edge of the hill and watch to see where the ski jumpers land. They decide the distances with their eyes instead of measuring with any tools.

Answer key page 12, unit 240

Subtraction

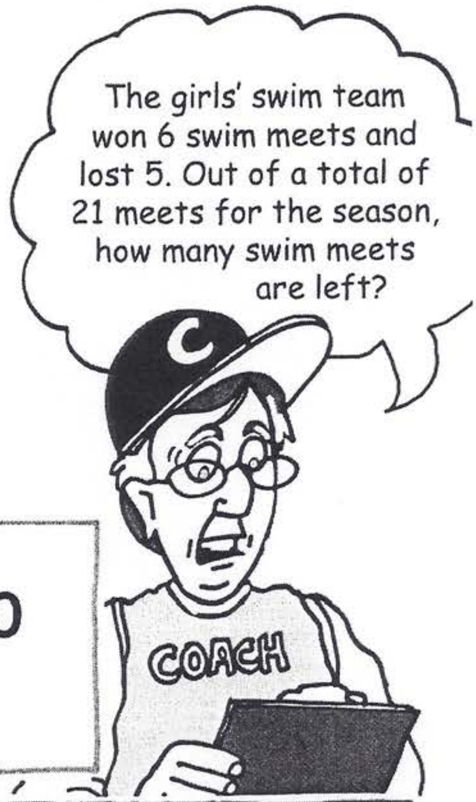
Subtraction is the operation of finding a missing addend (or, the taking away of one number or amount from another).

The symbol for subtraction is **—**

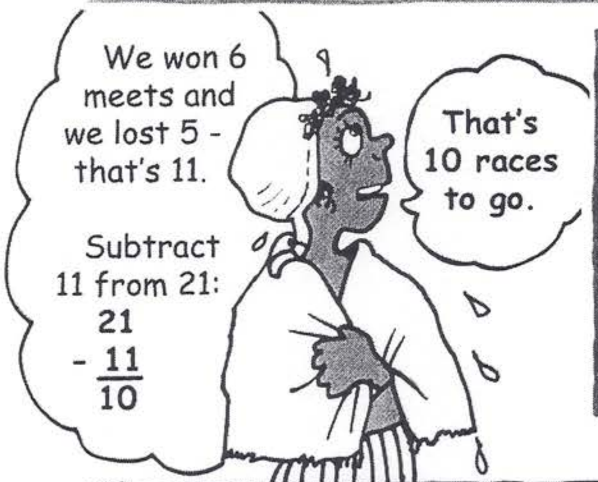
The word used for addition is ***minus***.

The number being subtracted from is the ***minuend***.

The number being subtracted is the ***subtrahend***.



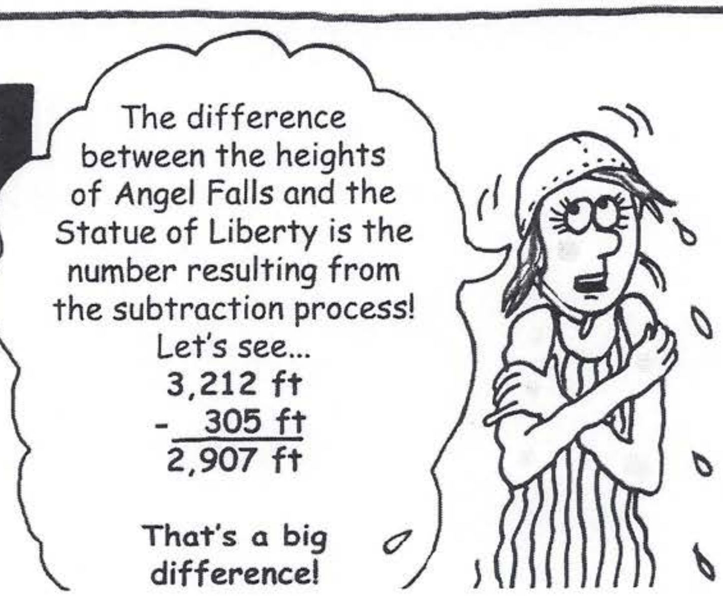
$$\underset{\text{minuend}}{505{,}000} - \underset{\text{subtrahend}}{5{,}000} = \underset{\text{difference}}{500{,}000}$$



8	minuend	988
- 3	subtrahend	- 237
5	difference	751

WHATS THE DIFFERENCE?

Angel Falls in Venezuela is the world's tallest waterfall. It is 3,212 feet tall. The Statue of Liberty is 305 feet tall. What's the difference between their heights?



Subtraction with Borrowing (or Renaming)

Sometimes a digit in the minuend is smaller than the digit of the same place in the subtrahend. When this happens, it is necessary to ***borrow*** from the next place to the left.

Borrowing is the same as ***renaming***. It means exchanging a ten to make a number in the ones place **larger** than the digit in the subtrahend. (OR, it might mean exchanging a hundred for 10 tens, or a thousand for 10 hundreds, etc.)

Addition & Subtraction Are Relatives!

Addition and subtraction are opposite (inverse) operations.

$$9 + 7 = 16$$
$$16 - 9 = 7$$
and $16 - 7 = 9$

An addition problem can be checked with subtraction.

$$\begin{array}{r} 8{,}222 \\ +\ 9{,}666 \\ \hline 17{,}888 \end{array} \qquad \begin{array}{r} 17{,}888 \\ -\ 9{,}666 \\ \hline 8{,}222 \end{array}$$

A subtraction problem can be checked with addition.

$$\begin{array}{r} 50{,}000 \\ -\ \ \ 500 \\ \hline 49{,}500 \end{array} \qquad \begin{array}{r} 49{,}500 \\ +\ \ \ 500 \\ \hline 50{,}000 \end{array}$$

In this example, 8 is too large to subtract from 5. So, one of the tens is ***borrowed or renamed*** as 10 ones.

Now there are 15 ones.
8 can be subtracted from 15.

That leaves only 2 tens.
(See the 2 written above the tens place.)

tens	ones
$\overset{2}{\cancel{3}}$	$\overset{1}{5}$
$-$	8
2	7

In this, the 8 in the tens place is smaller than the 9 in the tens place. So, one of the hundreds is ***borrowed or renamed*** as 10 tens.

Now there are 18 tens.
9 can easily be subtracted from 18.

This leaves only 6 hundreds.
(See the 6 written above the hundreds place.)

hundreds	tens	ones
$\overset{6}{\cancel{7}}$	$\overset{1}{8}$	5
$-$	9	3
6	8	2

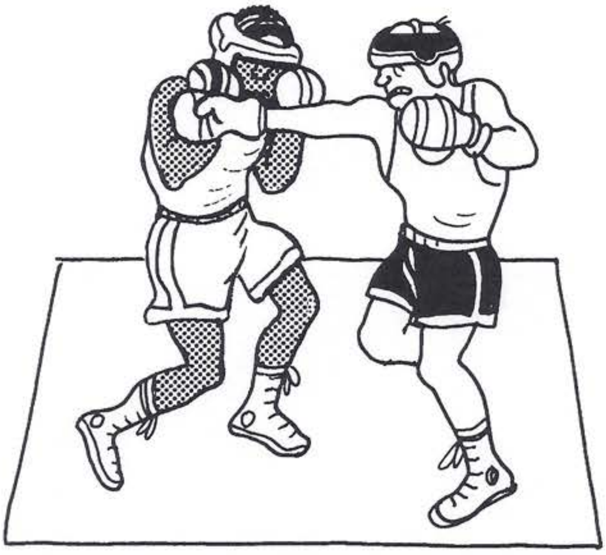
Get Sharp Tip #8
Renaming is also called **regrouping.**

KNOCK OUT!

Boxing was not allowed at the first modern Olympics in 1896 because it was considered too ungentlemanly and dangerous. Today it is a very popular Olympic sport. Some of the world's greatest boxers, such as Floyd Patterson, Muhammad Ali, Sugar Ray Leonard, Joe Frazier, Leon Spinks, and Evander Holyfield, won Olympic medals before becoming professional boxers.

See if you can knock out these subtraction problems by getting all the answers right!

1. $500 - 229$
2. $900 - 683$
3. $40 - 26$
4. $300 - 258$



5. $407 - 133$
6. $90 - 55$
7. $800 - 393$
8. $5500 - 203$
9. $9050 - 5348$
10. $7001 - 6420$
11. $6110 - 456$
12. $8006 - 731$
13. $800{,}321 - 79{,}001$
14. $32{,}000 - 19{,}862$

Olympic Fact

One of the most memorable moments of the 1996 Summer Olympic Games in Atlanta was when boxing legend and 1960 gold medal-winner Muhammad Ali (who suffers from Parkinson's disease) lit the Olympic torch.

LEARNING THROUGH SPORTS™

Mathematics:
Fractions

The following section of this customized textbook includes material from these skill areas:

Skill Description

2033: compare fractions

4.NF.2: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

2035: divide regions into equal parts and recognize equal and unequal regions

4.NF.1: Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalentfractions.

2036: explore fractions using a variety of representations

4.NF.5: Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.

4.NF.1: Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalentfractions.

2037: identify and represent equivalent fractions

4.NF.1: Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalentfractions.

2038: identify fractions

4.NF.5: Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.

4.NF.1: Explain why a fraction a/b is equivalent to a fraction (n x a)/(n x b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalentfractions.

2039: identify mixed numbers

4.NF.3.c: Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

4.NF.2: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

2044: relate fractions to decimals

4.NF.6: Use decimal notation for fractions with denominators 10 or 100.

4.NF.3.b: Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.

2045: solve problems with fractions

4.NF.2: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Comparing & Ordering Fractions

Sometimes you can look at two fractions and know for sure that one is smaller or larger than the other. At other times, you may not be sure.

Here are some ways to find out exactly how two fractions compare.

$\frac{1}{3}$ and $\frac{4}{9}$

$1 \times 9 = 9$

$3 \times 4 = 12$

$9 < 12$

$\frac{1}{3} < \frac{4}{9}$

How to Compare & Order Two Fractions

To compare $\frac{1}{3}$ and $\frac{4}{9}$, cross multiply the two fractions.

Step 1: Multiply the first numerator and the second denominator: $1 \times 9 = 9$

Step 2: Multiply the first denominator and the second numerator: $3 \times 4 = 12$

Step 3: Compare products:
If the first multiplication has the greater product, the first fraction is greater.
If the second multiplication has the greater product, the second fraction is greater.

9 is less than 12, therefore $\frac{1}{3} < \frac{4}{9}$

Get Sharp # 14

To compare mixed fractional numbers, first change them into improper fractions. Then follow either of the methods for comparing.

How to Compare & Order Several Fractions

Change all the fractions to like fractions (common denominators). Then it is easy to put them in order.

$1\frac{2}{3} \qquad \frac{2}{5} \qquad \frac{5}{6} \qquad \frac{7}{15}$

(The least common denominator is 30.)

$1\frac{2}{3} = \frac{5}{3} = \frac{50}{30} \qquad \frac{2}{5} = \frac{12}{30} \qquad \frac{5}{6} = \frac{25}{30}$

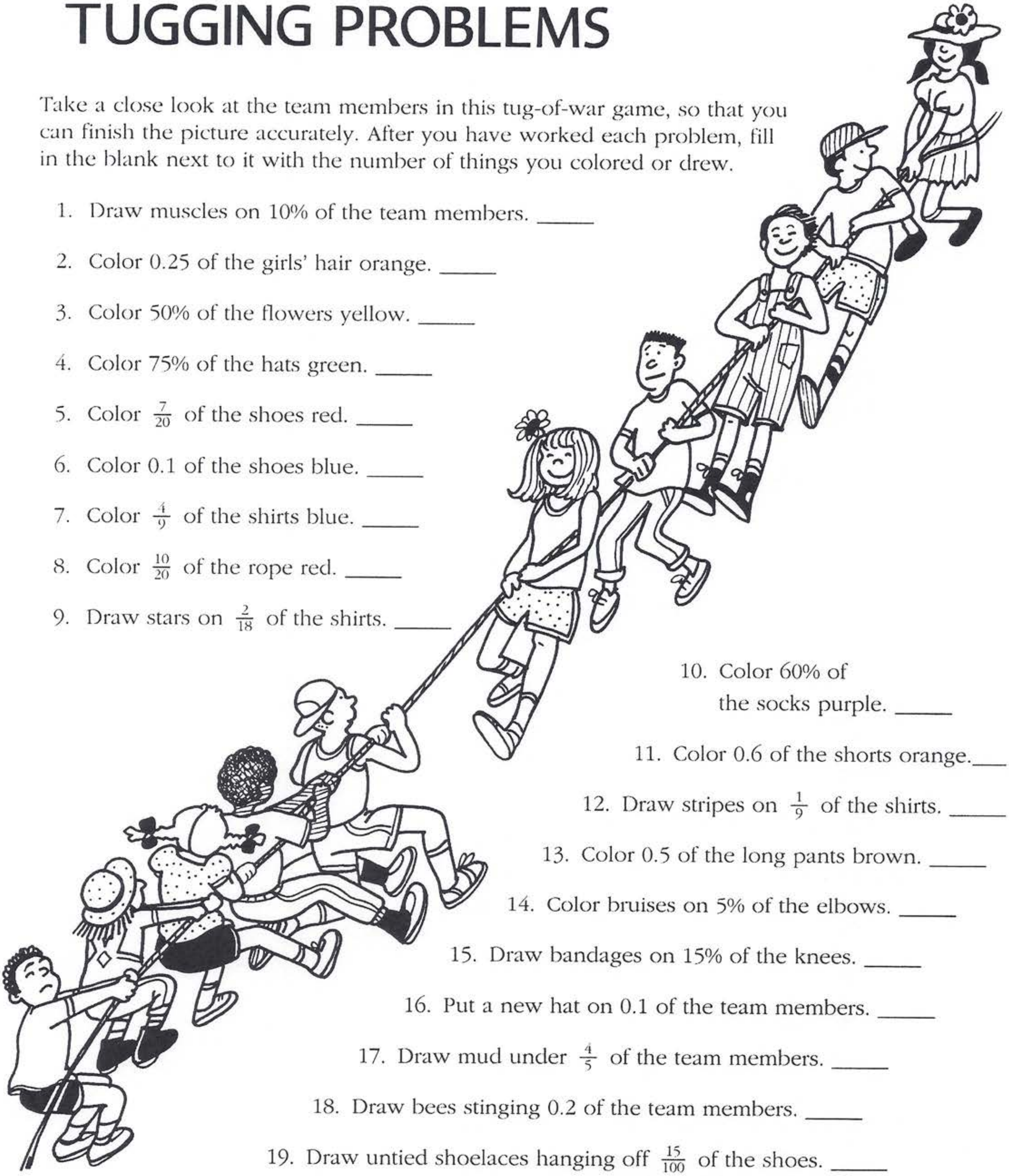
$\frac{7}{15} = \frac{14}{30}$

The right order is:

$\frac{2}{5} \ldots \frac{7}{15} \ldots \frac{5}{6} \ldots 1\frac{2}{3}$

TUGGING PROBLEMS

Take a close look at the team members in this tug-of-war game, so that you can finish the picture accurately. After you have worked each problem, fill in the blank next to it with the number of things you colored or drew.



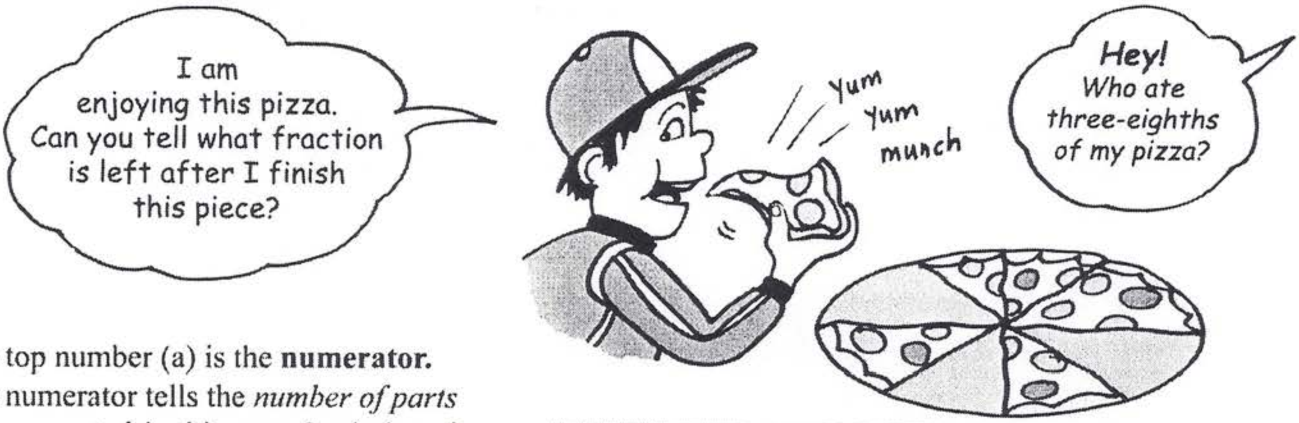
1. Draw muscles on 10% of the team members. _____
2. Color 0.25 of the girls' hair orange. _____
3. Color 50% of the flowers yellow. _____
4. Color 75% of the hats green. _____
5. Color $\frac{7}{20}$ of the shoes red. _____
6. Color 0.1 of the shoes blue. _____
7. Color $\frac{4}{9}$ of the shirts blue. _____
8. Color $\frac{10}{20}$ of the rope red. _____
9. Draw stars on $\frac{2}{18}$ of the shirts. _____
10. Color 60% of the socks purple. _____
11. Color 0.6 of the shorts orange.___
12. Draw stripes on $\frac{1}{9}$ of the shirts. _____
13. Color 0.5 of the long pants brown. _____
14. Color bruises on 5% of the elbows. _____
15. Draw bandages on 15% of the knees. _____
16. Put a new hat on 0.1 of the team members. _____
17. Draw mud under $\frac{4}{5}$ of the team members. _____
18. Draw bees stinging 0.2 of the team members. _____
19. Draw untied shoelaces hanging off $\frac{15}{100}$ of the shoes. _____
20. Draw a dog pulling on the shirt of $\frac{10}{100}$ of the team members. _____

Fractions

A **fraction** is any number written in the form of $\frac{a}{b}$

FRACTION ACTION

Fraction comes from the Latin word *fractio*, meaning *broken parts*. ***Fraction*** means ***part of a set*** or ***part of a whole***. A fraction is written in a way that compares two numbers or amounts.



The top number (a) is the **numerator.**
The numerator tells the *number of parts being counted,* in this case, 3 missing pieces.

Write the fraction like this:

3	(a) *missing pieces*
8	(b) *pieces in the whole pizza*

The bottom number (b) is the **denominator.**
The denominator tells *the number of parts in the whole,* or 8 pieces of pizza.

Proper & Improper Fractions

In a **proper fraction**, the numerator is *smaller* than the denominator. $\frac{2}{9}$

In an **improper fraction**, the numerator is *larger* than the denominator. The value of the fraction is always *equal to* or *greater than* one. $\frac{12}{7}$

$\frac{7}{8}$ reads ***seven-eighths***

$\frac{11}{12}$ reads ***eleven-twelfths***

$\frac{2}{3}$ reads ***two-thirds***

$\frac{14}{20}$ reads ***fourteen-twentieths***

$\frac{3}{100}$ reads ***three-hundredths***

$\frac{6}{9}$ reads ***six-ninths***

Reading and Writing Fractions

A fraction is also a way of writing a division problem.

$\frac{3}{24}$ means $3 \div 24$

(three divided by twenty-four)

Fractions as Part of a Whole

Some fractions represent parts of a whole. Each pizza is a whole item, cut into parts.

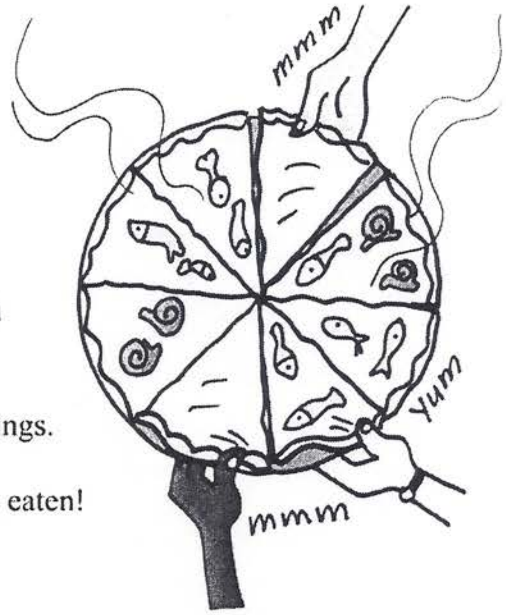
> **Get Sharp Tip #12**
> The denominator in a fraction cannot be zero.

$\frac{5}{8}$ of the pizza is topped with anchovies.

$\frac{2}{8}$ of the pizza is topped with snails.

$\frac{1}{8}$ of the pizza has both toppings.

$\frac{3}{8}$ of this pizza is about to be eaten!



Fractions as Part of a Set

Sometimes, a fraction represents part of a set. This picture includes a set of football fans. It also includes a set of blankets, feet, banners, other sets. The fractions answer questions about the sets.

How many fans are wearing **hats**?.............................. $\frac{4}{5}$ ....(the set: fans)

How many blankets have **stripes**? .... $\frac{1}{3}$ ....(the set: blankets)

How many feet are wearing **athletic shoes**? .... $\frac{7}{10}$ ....(the set: feet)

How many fans are carrying **thermoses**?.................. $\frac{2}{5}$ ....(the set: fans)

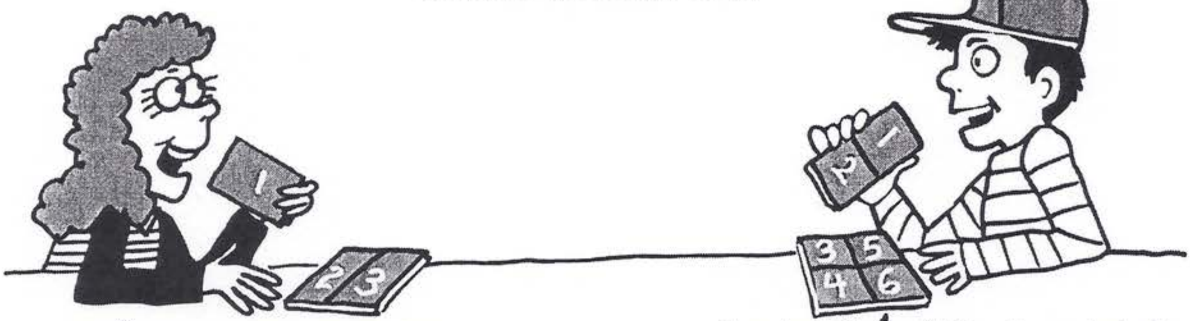
How many banners are for the **Grizzlies**?.................. $\frac{2}{3}$ ....(the set: banners)

How many of the hands **are bare**?.............................. $\frac{4}{10}$ ....(the set: hands)

Equivalent Fractions

Equivalent fractions are two or more fractions that represent the same amount.

Two friends start out with chocolate bars that are the same size.



B.J. left $\frac{2}{3}$ of her chocolate bar on the table.

Louie left $\frac{4}{6}$ of his chocolate bar on the table.

$\frac{2}{3}$ and $\frac{4}{6}$ are equivalent. Write: $\frac{2}{3} = \frac{4}{6}$.

How to Form Equivalent Fractions

Step 1: Multiply or divide both the numerator and the denominator by the same nonzero number.

Step 2: Write the new fraction.

$$\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$$

$$\frac{56}{72} = \frac{56 \div 8}{72 \div 8} = \frac{7}{9}$$

How to Tell Equivalent Fractions

Step 1: Cross multiply.

Step 2: Compare the two products.

Step 3: If the products are equal, the fractions are equivalent. Otherwise they are not.

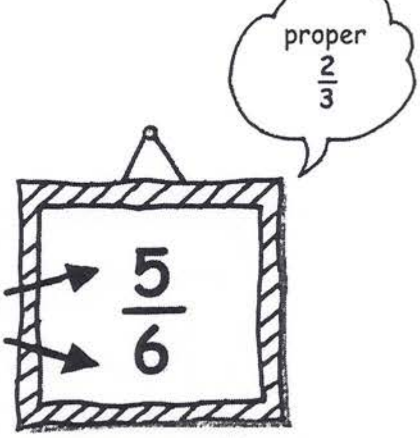
$\frac{2}{5}$ $\frac{4}{10}$ → $2 \times 10 = 20$, $5 \times 4 = 20$ → **20 = 20,** ***so the fractions are equivalent***

$\frac{7}{9}$ $\frac{4}{5}$ → $7 \times 5 = 35$, $9 \times 4 = 36$ → **35 ≠ 36,** ***so the fractions are not equivalent***

Fractions

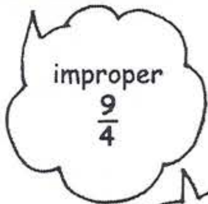
Fraction means **part of a whole or part of a set.**

The set of **fractions** includes any number in the form of $\frac{a}{b}$ that compares part of an object or part of a set to the whole.
(The bottom number (b) cannot be 0.)



A fractional number has two parts: a **numerator** (top number) and a **denominator** (bottom number).

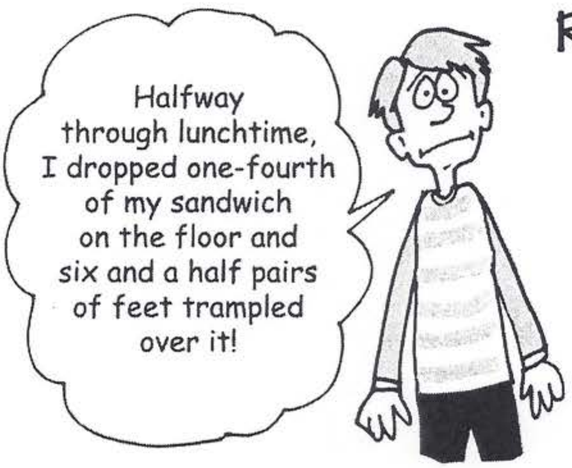
The **numerator** tells the number of parts that are being counted.
The **denominator** tells the number of parts in the whole.



In a **proper fraction,** the numerator is smaller than the denominator.

In an **improper fraction,** the numerator is greater than the denominator.

A **mixed fractional numeral** contains a whole number and a fraction.



Reading and Writing Fractions

$\frac{2}{3}$ reads ***two-thirds***

$\frac{4}{6}$ reads ***four-sixths***

$\frac{7}{8}$ reads ***seven-eighths***

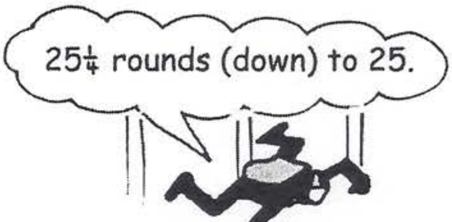
$\frac{13}{9}$ reads ***thirteen-ninths***

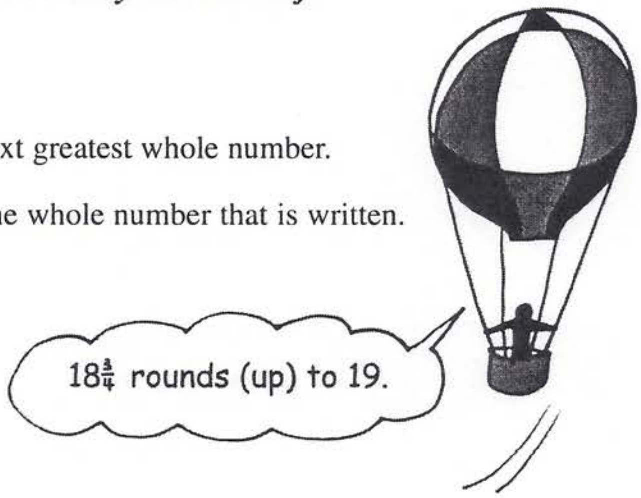
$20\frac{1}{2}$ reads ***twenty and one-half***

Rounding Mixed Numbers

If the fraction **is worth $\frac{1}{2}$ or more**, round up to the next greatest whole number.

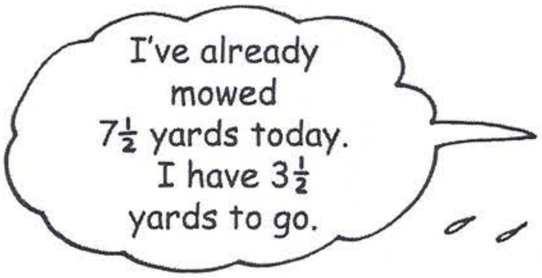
If the fraction **is worth less than $\frac{1}{2}$**, round down to the whole number that is written.



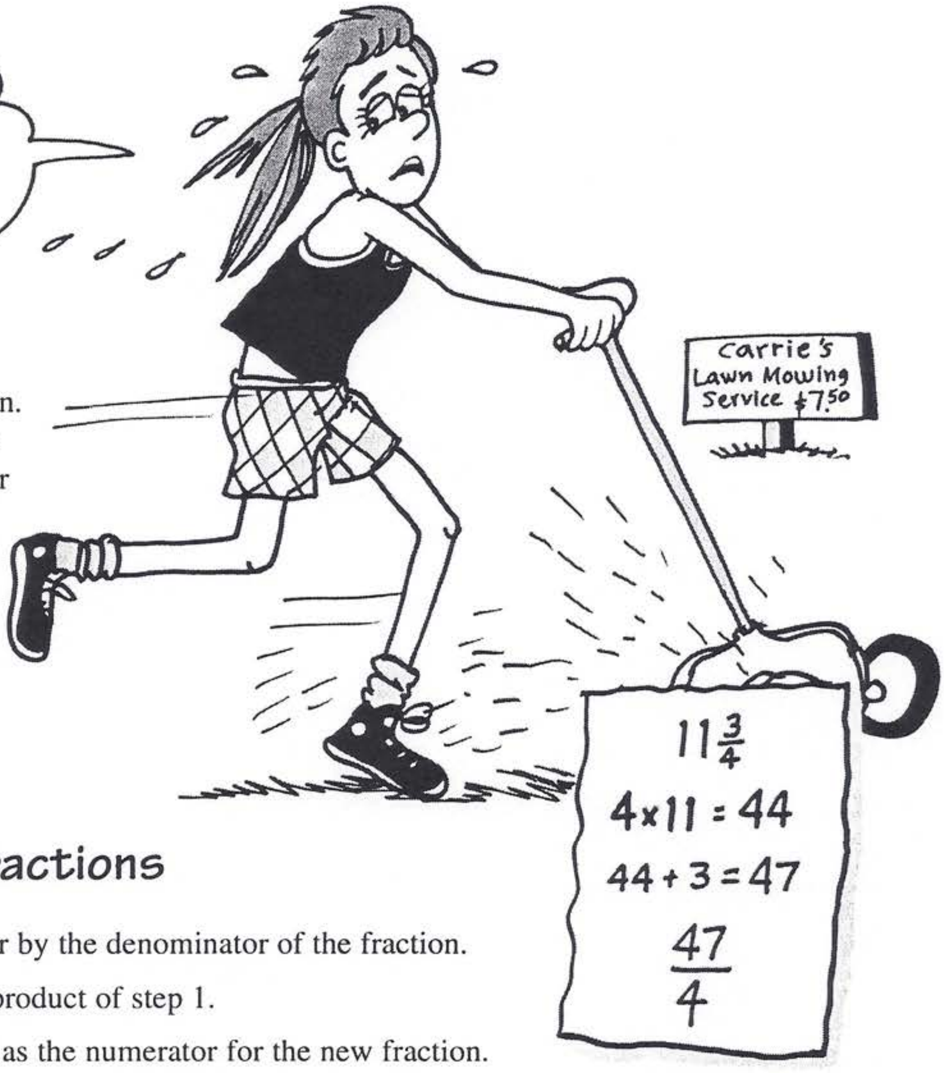


Mixed Fractional Numbers

A **mixed fractional number** combines a whole number and a fraction. The value of a mixed fractional number is always greater than one (unless it is a negative number).



A mixed fractional number may be written as an improper fraction. Sometimes it is useful to change a mixed number into an improper fraction in order to complete an operation.



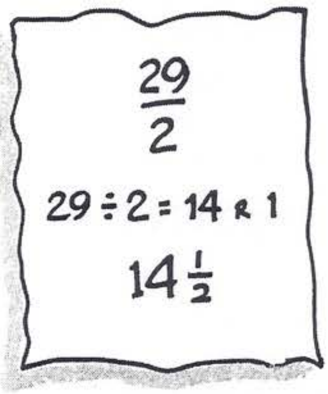
How to Change Mixed Numbers to Improper Fractions

Step 1: Multiply the whole number by the denominator of the fraction.

Step 2: Add the numerator to the product of step 1.

Step 3: Write the sum from step 2 as the numerator for the new fraction.

Step 4: Write the original denominator as the denominator for the new fraction.



How to Change Improper Fractions to Mixed Numbers

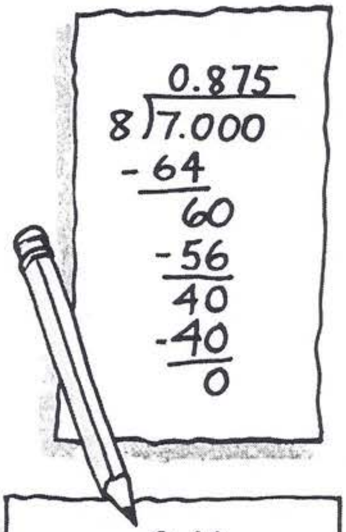
Step 1: Divide the numerator by the denominator.

Step 2: Write the quotient as a whole number.

Step 3: Write the remainder as the numerator in a fraction.

Step 4: Write the original denominator in the fraction.

Fractions & Decimals



0.44
= *forty-four hundredths*
$= \frac{44}{100}$
$= \frac{11}{25}$

How to Write a Fraction as a Decimal

Step 1: Divide the numerator by the denominator.

Step 2: Write a zero to hold the ones place (if there is no number in that place).

$$\frac{7}{8} = 0.875$$

How to Write a Decimal as a Fraction

Step 1: Remove the decimal point and write the number as the numerator. The denominator is 10 or a multiple of 10, depending what place the last digit of the decimal occupied. For instance, in 0.044, the last digit is a thousandth.

Step 2: Reduce the fraction to lowest terms.

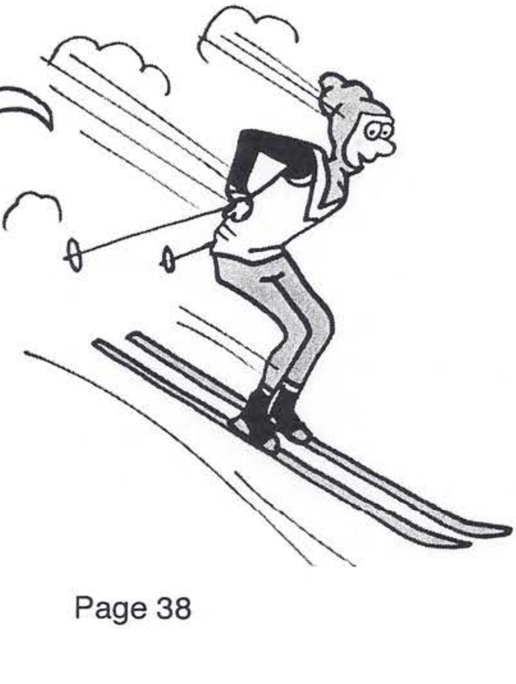
$\frac{44}{1000}$ reduced to lowest terms is $\frac{11}{250}$.

WHO WON?

Ari skied the run in 7.38 minutes.	Ramon's time was $7\frac{3}{4}$ minutes.	Danny's time was 0.15 hours.

To solve a problem that has some terms in decimals and others in fractions, change everything to decimals. Then compare the amounts.

7.38 min. = 7.38 min.
$7\frac{3}{4}$ min. = 7.75 min.
0.15 hours = 0.15 x 60 = 9 min.



Ari is the fastest.

Multiplying Fractions

The top running speed of a human is less than $\frac{1}{2}$ the speed of an ostrich.

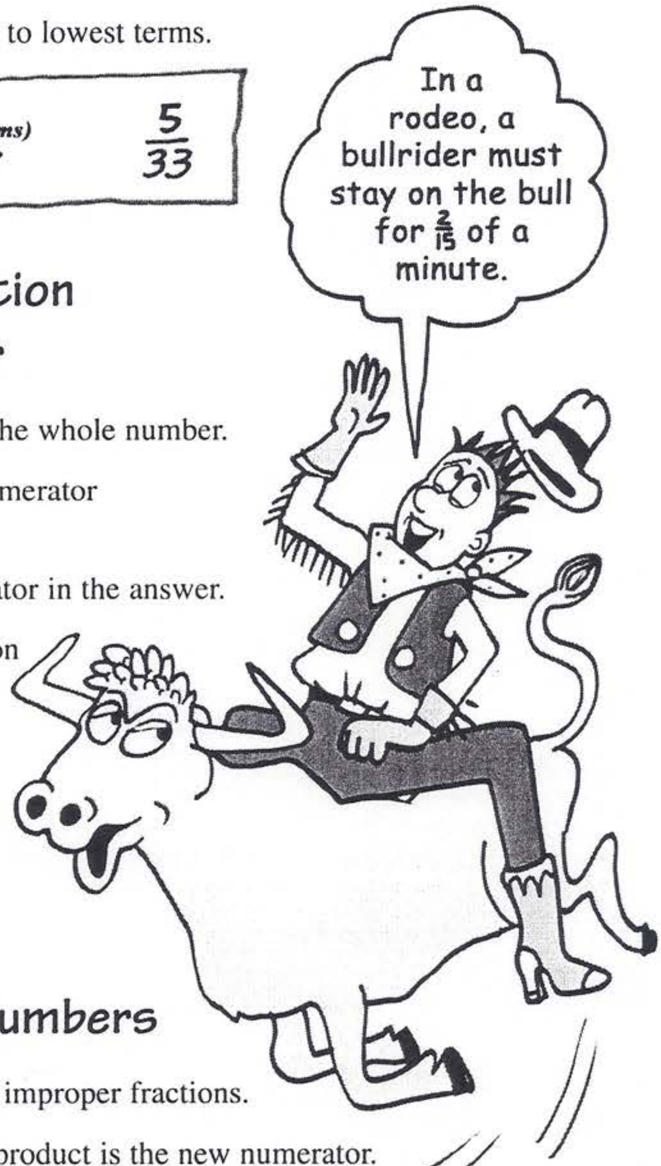
How to Multiply Fractions

Step 1: Multiply the numerators; this product is the new numerator.

Step 2: Multiply the denominators; this product is the new denominator.

Step 3: Reduce the product fraction to lowest terms.

$$\frac{5}{9} \times \frac{3}{11} = \frac{15}{99} \xrightarrow{\textit{(in lowest terms)}} \frac{5}{33}$$



No matter how big an iceberg is, $\frac{8}{9}$ of it will hide below the surface of the water.

How to Multiply a Fraction by a Whole Number

Step 1: Multiply the numerator by the whole number.

Step 2: Write this product as the numerator in the answer.

Step 3: Write the original denominator in the answer.

Step 4: Change the improper fraction into a mixed numeral, and reduce to lowest terms.

$$4 \times \frac{2}{3} = \frac{8}{3} = 2\frac{2}{3}$$

The *Titanic* was $882\frac{3}{4}$ feet long.

How to Multiply Mixed Numbers

Only $\frac{1}{3}$ of all humans can flare their nostrils.

Step 1: Change all mixed numerals to improper fractions.

Step 2: Multiply the numerators; this product is the new numerator.

Step 3: Multiply the denominators; this product is the new denominator.

Step 4: Change the improper fraction into a mixed numeral, and reduce to lowest terms.

$$7\frac{1}{2} \times 5\frac{2}{5} = \frac{15}{2} \times \frac{27}{5} = \frac{405}{10} = 40\frac{5}{10} = 40\frac{1}{2}$$

Dividing Fractions

How to Divide Fractions

Step 1: Invert (flip over) the second fraction (the divisor fraction).

Step 2: Change the problem into a multiplication problem.

Step 3: Multiply the fractions.

Step 4: Reduce the quotient fraction to lowest terms.

$$\boxed{\frac{1}{4} \div \frac{2}{3} = \frac{1}{4} \times \frac{3}{2} = \frac{3}{8}}$$

How to Divide a Whole Number by a Fraction
(or a Fraction by a Whole Number)

Step 1: Change the whole number into an improper fraction with the whole number as the numerator and 1 as the denominator.

Step 2: Proceed with the instructions for dividing fractions.

Step 3: Change any improper fractions in the quotient to mixed numerals, and reduce to lowest terms.

$$\boxed{5 \div \frac{2}{5} = \frac{5}{1} \div \frac{2}{5} = \frac{5}{1} \times \frac{5}{2} = \frac{25}{2} = 12\frac{1}{2}}$$

How to Divide Mixed Numbers

Step 1: Change any mixed numbers into improper fractions.

Step 2: Proceed with the instructions for dividing fractions.

Step 3: Change any improper fractions in the quotient to mixed numerals, and reduce to lowest terms.

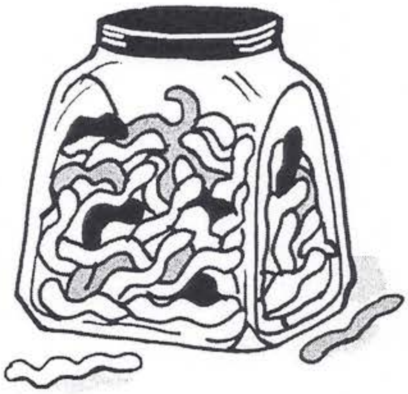
$$\boxed{2\frac{1}{2} \div 4\frac{1}{5} = \frac{5}{2} \div \frac{21}{5} = \frac{5}{2} \times \frac{5}{21} = \frac{25}{42}}$$

Problem-Solving Strategies

A problem-solving strategy is a method for approaching and solving a problem. There are many different ways to solve problems. Different strategies fit well with different kinds of problems.

One of the skills involved in sharp problem solving is being able to choose a good strategy. Here are some strategies to have among your list of tools for attacking problems. (See pages 196–206).

Guess & Check



Sometimes the best strategy for solving a problem is to make a smart guess. After you make a careful guess, if it is possible, you can count or calculate to see if your guess was right.

The **Guess & Check** strategy is a good one for a problem where you can see a quantity, but it is too large, complex, or far away to count accurately and easily. Use it for this problem.

The Problem:

How many candy worms are in this jar?

Trial & Error

For some problems, the best strategy is to try different solutions until you find one that works.

Trial & Error is a good strategy for those tricky age problems.

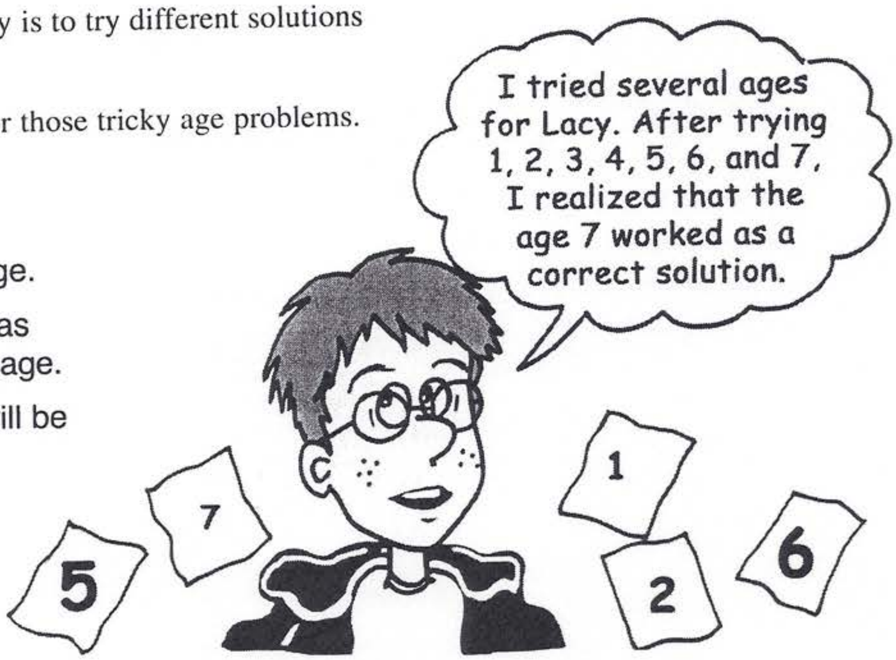
The Problem:

Tracy is twice Lacy's age.

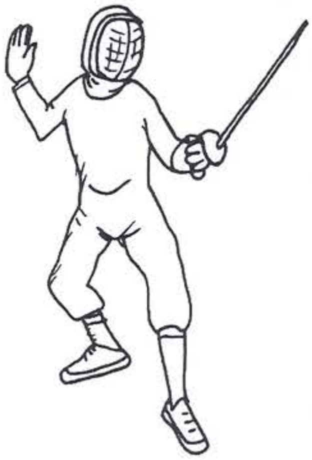
Six years ago, Tracy was eight times Lacy's age.

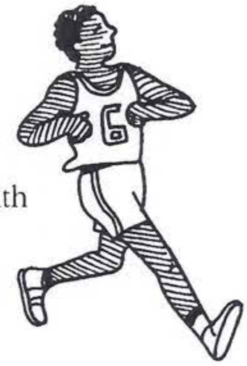
In seven years, Lacy will be $\frac{2}{3}$ of Tracy's age.

How old is Lacy?



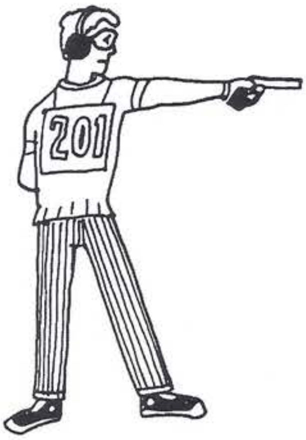
PENTATHLON CALCULATIONS





Penta means five, so athletes who compete in the pentathlon have to be good at five different sports. The modern pentathlon is based on the duties of a warrior who must deliver a message across enemy lines. He has to ride a horse around many obstacles, defend himself with a sword and gun, run great distances, and swim across rivers and streams. Olympic competitors must complete contests in equestrian riding, fencing, pistol shooting, running, and swimming.

Each of these calculations has five parts, also. You need to be good at each step in order to get the right answer!



1. $\frac{9}{10} - \frac{1}{10} + \frac{7}{10} + \frac{5}{10} - \frac{1}{10} =$ ____________

2. $\frac{7}{9} - \frac{3}{9} - \frac{2}{9} + \frac{6}{9} - \frac{2}{9} =$ ____________

3. $\frac{2}{13} + \frac{5}{13} - \frac{4}{13} + \frac{10}{13} - \frac{6}{13} =$ ____________

4. $\frac{5}{6} - \frac{2}{6} + \frac{6}{6} - \frac{2}{6} - \frac{3}{6} =$ ____________

5. $\frac{3}{20} - \frac{1}{20} + \frac{15}{20} - \frac{4}{20} + \frac{2}{20} =$ ____________

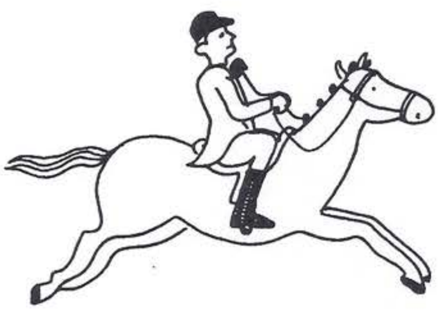
6. $\frac{1}{11} + \frac{5}{11} + \frac{8}{11} - \frac{9}{11} + \frac{1}{11} =$ ____________

7. $\frac{1}{5} - \frac{1}{5} + \frac{2}{5} + \frac{9}{5} - \frac{6}{5} =$ ____________

8. $\frac{4}{16} - \frac{2}{16} + \frac{7}{16} + \frac{5}{16} + \frac{1}{16} =$ ____________

9. $\frac{5}{25} - \frac{3}{25} + \frac{9}{25} - \frac{2}{25} + \frac{1}{25} =$ ____________

10. $\frac{6}{12} - \frac{5}{12} + \frac{7}{12} - \frac{7}{12} + \frac{1}{12} =$ ____________



11. $\frac{9}{6} - \frac{2}{6} + \frac{10}{6} - \frac{3}{6} + \frac{15}{6} =$ ____________

12. $\frac{11}{100} + \frac{15}{100} - \frac{4}{100} + \frac{50}{100} - \frac{1}{100} =$ ____________

13. $\frac{3}{30} + \frac{8}{30} + \frac{14}{30} - \frac{7}{30} - \frac{2}{30} =$ ____________

14. $999\frac{8}{10} - 999 + \frac{14}{10} - \frac{2}{10} + \frac{5}{10} =$ ____________

LEARNING THROUGH SPORTS™

Mathematics:
Decimals

The following section of this customized textbook includes material from these skill areas:

Skill Description

2023: compare decimals

4.NF.7: Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

2025: explore decimals using a variety of representations

4.NF.7: Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

2027: order decimals

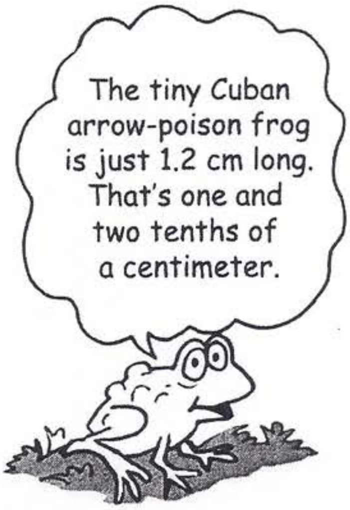
4.NF.7: Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

2028: read decimals

2031: solve problems with decimals

4.NF.7: Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Place Value in Decimals



Places to the right of the ones place show decimals.
A decimal point separates the ones place from the tenths place.

The chart below shows the first six places to the right of the decimal point.

tens	ones	tenths	hundredths	thousandths	ten thousandths	hundred thousandths	millionths
	5.	5					
	1.	1	2	3			
	0.	0	0	7	1		
	0.	1	5	0	5	5	
1	2.	0	0	0	8	6	6

Reading & Writing Decimals

Read the whole number first. Then, read the entire number to the right of the decimal point, adding the label from the place of the last digit.

5.5 reads *five and five tenths*

1.123 reads *one and one hundred twenty-three thousandths*

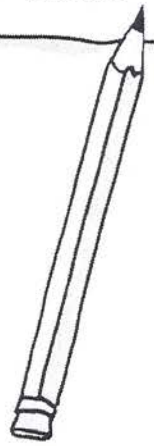
0.0071 reads *seventy-one ten thousandths*

0.15055 reads *fifteen thousand fifty-five hundred thousandths*

12.000866 reads *twelve and eight hundred sixty-six millionths*

Writing Decimals

nine thousandths
= 0.009
nine millionths
= 0.000009
nine hundreths
= 0.09
nine ten thousandths
= 0.0009



Rounding Decimals

Decimals are rounded in the same way as whole numbers.

If a digit is 5 or greater, round up to the next highest value in the place to the left. If the digit is 4 or less, round down.

0.005 *rounded to the nearest hundred is* 0.01

0.63 *rounded to the nearest tenth is* 0.6

5.068 *rounded to the nearest tenth is* 5.1

5.068 *rounded to the nearest hundredth is* 5.07

NOSE ROLLS & FAKIES

This must be the sport with the wildest names for moves and tricks! On a snowboard you can do Halfpipes, Nose Rolls, Wheelies, McTwists, Chicken Salads, and Ollies—and many more tricks with wild, wacky names! 1998 was the first time snowboarders could take part in the Olympic Games. The boarders were ready to do all these fancy tricks in Japan!

To finish each of these tricks with a good score, read the decimals on each card. Then number them in order from the largest to the smallest.

Trick #1 FAKIE

____	0.11103	____	1.7
____	0.103	____	11.3
____	10.37	____	13.01
____	11.370	____	0.13

Trick #2 NOSE ROLL

____	15.02	____	15.21
____	1.5	____	1.51
____	0.005	____	55.5
____	0.05	____	5.5

Trick #3 BACKSCRATCHER

____	4.5	____	4.7
____	0.451	____	44.5
____	0.45	____	4.4
____	0.06	____	0.44

Trick #4 McTWIST

____	5.28	____	5.6
____	9.97	____	0.009
____	0.8	____	5.8
____	0.99	____	0.08

Trick # 5 CHICKEN SALAD

____	2.6	____	6.2
____	2.7	____	2.9
____	2.06	____	22.6
____	26.6	____	2.999

Trick #6 OLLIE

____	7.2	____	7.7
____	0.72	____	77.27
____	0.072	____	0.07
____	72.1	____	0.007

Trick # 7 TAIL WHEELIE

____	0.0001	____	0.000001
____	0.001	____	0.01
____	101.1	____	10.11
____	1.1	____	0.00011

Decimals

The set of **decimals** includes all numbers in a base ten system. The term ***decimal,*** however, is often used to describe numbers that use a decimal point to show an amount between two whole numbers. A mixed decimal numeral is one that includes a whole number and digits to the right of the decimal point.

Some decimal numerals:

0.706 −18.5

10.0505

−6.07 127.3

Place Value in Decimals

Learn these places.

tenths hundredths thousandths ten thousandths hundred thousandths

5 . 1 2 3 4 5

5.1 .... reads ***five and one tenth.***

5.12 .... reads ***five and twelve hundredths.***

5.123 .... reads ***five and one hundred twenty-three thousandths.***

5.1234 .... reads ***five and one thousand two hundred thirty-four ten thousandths.***

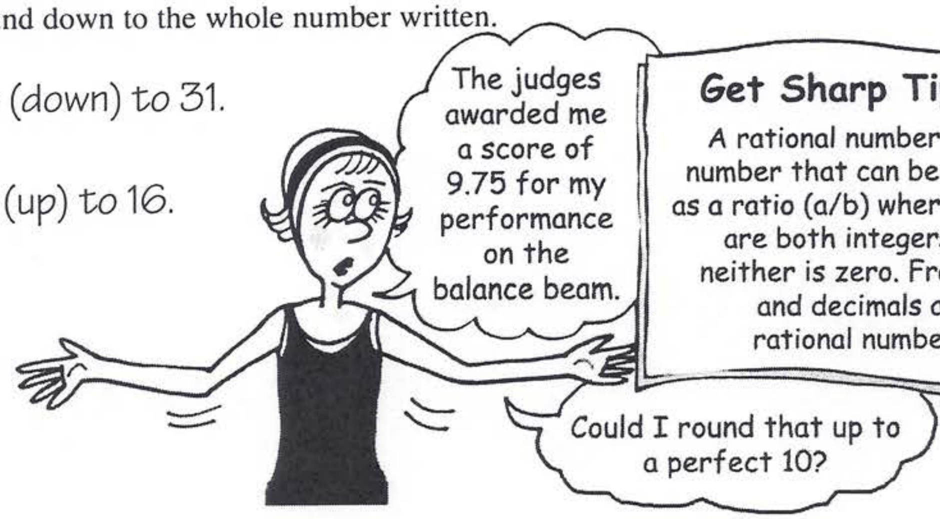
5.12345 .... reads ***five and twelve thousand three hundred forty-five hundred thousandths.***

Rounding Decimals

To round a mixed numeral to a whole number, look at the first digit to the right of the decimal point.
If it is 5 or greater, round up to the next whole number.
If it is less than 5, round down to the whole number written.

31.482 rounds (down) to 31.

15.677 rounds (up) to 16.

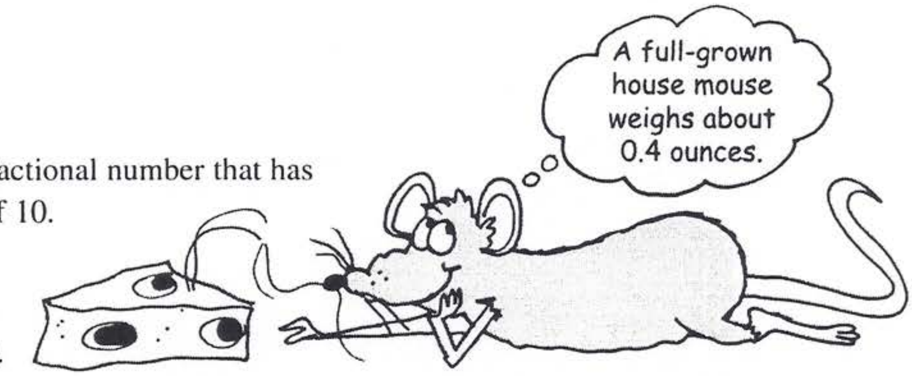


Get Sharp Tip #7

A rational number is any number that can be written as a ratio (a/b) where a and b are both integers and neither is zero. Fractions and decimals are rational numbers.

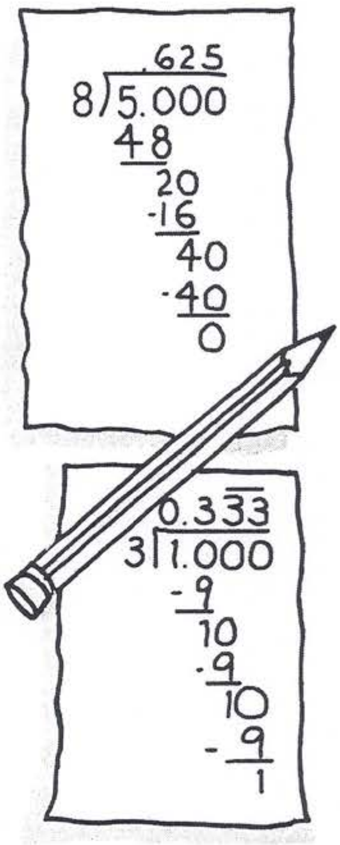
Decimals

A **decimal** is a way of writing a fractional number that has a denominator of 10 or a multiple of 10.



Decimals are written using a decimal point. The decimal point is placed to the right of the ones place.

$\frac{1}{10} = .1 \quad \frac{1}{100} = .01 \quad \frac{1}{1000} = .001 \quad \frac{1}{10{,}000} = .0001$



Terminating Decimals

A **terminating decimal** is a decimal number that ends. When a quotient for a divided fraction eventually shows a remainder of zero, the decimal terminates.

When $\frac{5}{8}$ is divided, the result is a terminating decimal.

Repeating Decimals

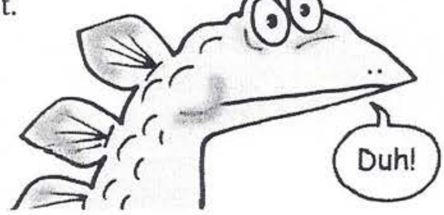
A **repeating decimal** is a decimal that has one or more digits that repeat indefinitely. The quotient for a divided fraction never results in a remainder of zero, and one or more of the final digits keep repeating. A repeating decimal is indicated by a bar written above the numbers that repeat.

When $\frac{1}{3}$ is divided, the result is a repeating decimal: $\mathbf{0.\overline{33}}$

Mixed Decimal Numbers

Mixed decimal numbers combine whole numbers and decimals.

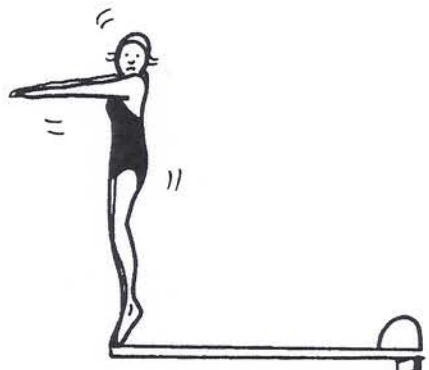
A mixed number has digits on both sides of the decimal point.



The average adult human brain weighs 1.4 kg. That's about 20 times the weight of the brain of a Stegosaurus.

Read Decimals

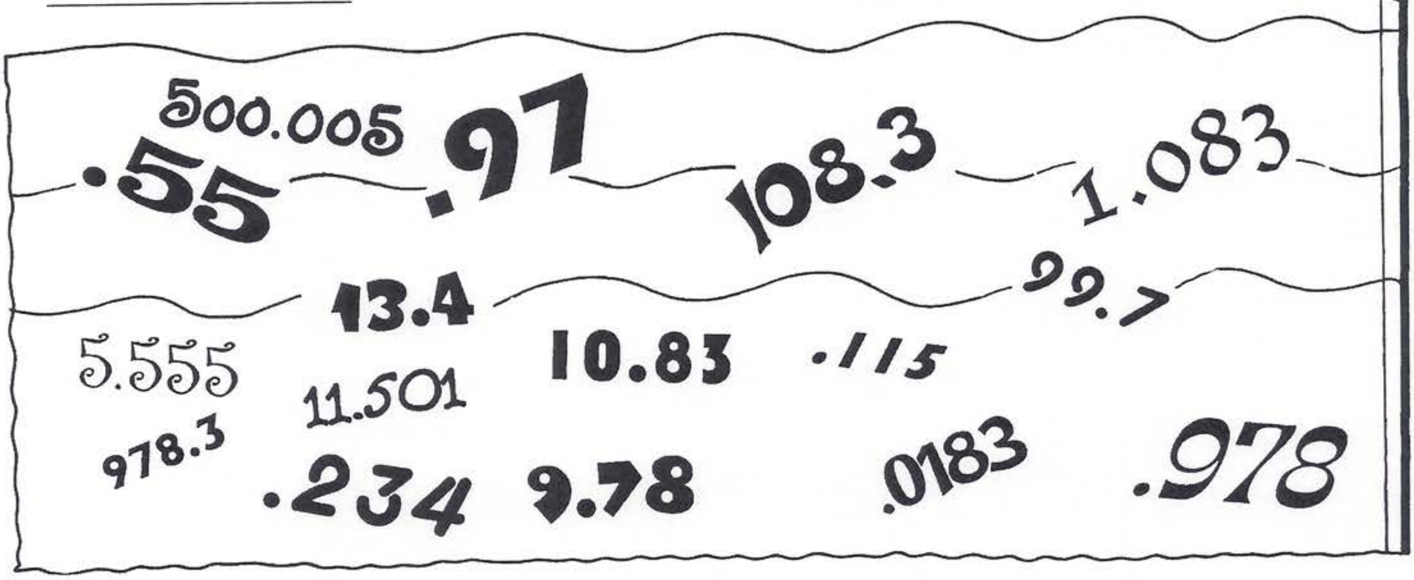
TAKE THE PLUNGE!



Can you imagine jumping off a three-story building into a pool of water? This is what platform divers do. Olympic divers either jump off high platforms, where they begin at a standstill, or they jump off a bouncy springboard. Seven judges watch each dive and score it between 0 and 10. Scores for eleven dives are added together. The diver with the highest score wins. In 1982, Mary Ellen Clark got a bronze medal for the USA with a score of 401.91.

Find a decimal in the pool to match each of the decimal words below.

1. one hundred eighty-three ten thousandths ____________
2. one and eighty-three thousandths ____________
3. one hundred fifteen thousandths ____________
4. fifty-five hundredths ____________
5. ninety-nine and seven tenths ____________
6. thirteen and four tenths ____________
7. five hundred and five thousandths ____________
8. nine and seventy-eight hundredths ____________
9. ninety-seven hundredths ____________
10. ten and eighty-three hundredths ____________
11. one hundred eight and three tenths ____________
12. two hundred thirty-four thousandths ____________
13. five and five hundred fifty-five thousandths ____________
14. nine hundred seventy-eight thousandths ____________
15. nine hundred seventy-eight and three tenths ____________
16. eleven and five hundred one thousandths ____________



Dividing a Decimal by a Whole Number

Step 1: Place the decimal point in the quotient directly above the decimal point in the dividend.

Step 2: Divide as you would with whole numbers.

Step 3: Add zeros where necessary to hold places.

Chuck climbed a giant Ponderosa Pine tree that was 68.4 feet tall. The climb took him four hours. On the average, how much distance did he climb each hour?

$$
\begin{array}{r}
17.1 \\
4\overline{)68.4} \\
-4 \\
\hline
28 \\
-28 \\
\hline
04 \\
-4 \\
\hline
0
\end{array}
$$

Charlene climbed an 81.9-foot Coast Douglas Fir tree. It was 2.6 times the height of the Western Red Cedar tree that C.J. climbed. How high was the cedar?

$$
\begin{array}{r}
31.5 \\
2.6\overline{)81.9} \\
-78 \\
\hline
39 \\
-26 \\
\hline
130 \\
-130 \\
\hline
0
\end{array}
$$

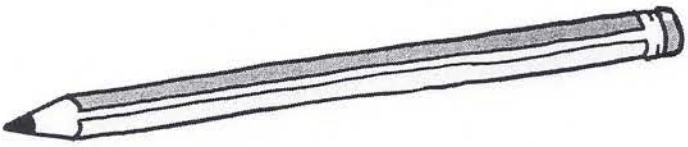
Dividing a Decimal by a Decimal

Step 1: Move the decimal point to the right to write the divisor as a whole number. Count the number of places you must move the decimal point.

Step 2: Move the decimal point in the dividend the same number of places to the right.

Step 3: Divide as you would with whole numbers.

Step 4: Align the decimal point in the quotient with the decimal point in the dividend.



LEARNING THROUGH SPORTS™

Mathematics:
Statistics

The following section of this customized textbook includes material from these skill areas:

Skill Description

2209: analyze data

4.MD.9: Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. NOTE: This is a reinforcement standard from the 3rd grade standard 3.MD.3.

2255: read and interpret charts and tables

4.MD.9: Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. NOTE: This is a reinforcement standard from the 3rd grade standard 3.MD.3.

2257: construct line graphs to represent data

4.MD.9: Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. NOTE: This is a reinforcement standard from the 3rd grade standard 3.MD.3.

2261: construct tree diagrams to represent data

4.MD.9: Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. NOTE: This is a reinforcement standard from the 3rd grade standard 3.MD.3.

2265: read and interpret bar graphs

4.MD.9: Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. NOTE: This is a reinforcement standard from the 3rd grade standard 3.MD.3.

2268: read and interpret line graphs

4.MD.9: Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. NOTE: This is a reinforcement standard from the 3rd grade standard 3.MD.3.

2271: read and interpret pictographs

4.MD.9: Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. NOTE: This is a reinforcement standard from the 3rd grade standard 3.MD.3.

2272: read and interpret tallies

4.MD.9: Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. NOTE: This is a reinforcement standard from the 3rd grade standard 3.MD.3.

2273: read and interpret Venn diagrams

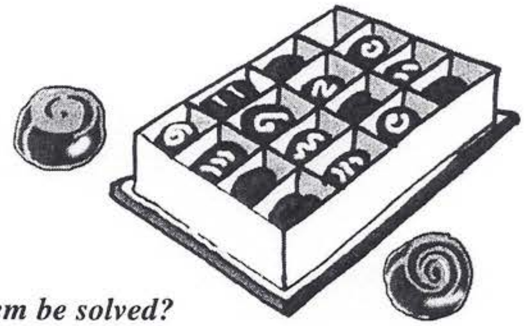
4.MD.9: Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. NOTE: This is a reinforcement standard from the 3rd grade standard 3.MD.3.

6420: define statistical terms

4.MD.9: Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. NOTE: This is a reinforcement standard from the 3rd grade standard 3.MD.3.

Approaching Problems

Defining the Problem



A **problem** is a question to be answered.
When you read a problem, try to identify the problem clearly.

Ask yourself:

What is the exact question to be answered? Can the problem be solved?

> The longest bone in the human body is the femur (thighbone). The humerus (the upper arm bone) is 5.53 in. shorter than the femur. How long is the humerus?

The question is:
How long is the humerus?

There is not enough information to solve this problem.

> In Switzerland, people consume an average of 26 pounds of chocolate each year. This is 2 pounds more than twice the consumption of the average Australian. What is the per person chocolate consumption in Australia?

The question is:
What is the per person chocolate consumption in Australia?

This problem can be solved.

Examining the Information

Identify the information needed to solve the problem. Sometimes there is more than you need. Sometimes information is missing.

> In Denmark, people chew more gum than anywhere else in the world. They chew three times as much gum as people in Japan. On the average, how much do the Japanese chew each year?

There is not enough information.

To solve the problem, you need to know how much gum is chewed in Denmark.

> In Norway, 2.6 kg of potato chips are eaten per person per year. Norwegians eat more frozen food than any other country. Swedes eat 1.6 kg of potato chips (per person) each year. How much greater is the amount of potato chips eaten by the average Norwegian?

There is too much information.

You do not need to know the frozen food consumption in order to solve the problem.

Make a Table or Graph

If there is a lot of data to consider in solving a problem, try putting it into a table. That way you can easily see relationships between numbers. A graph can also help you see relationships between numbers.

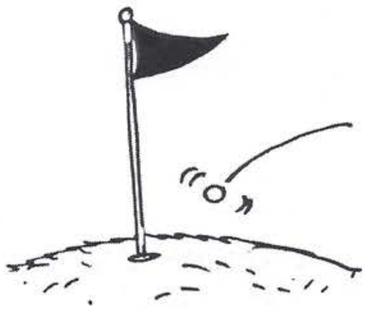
A line graph is especially helpful if you need to see how amounts change over time.

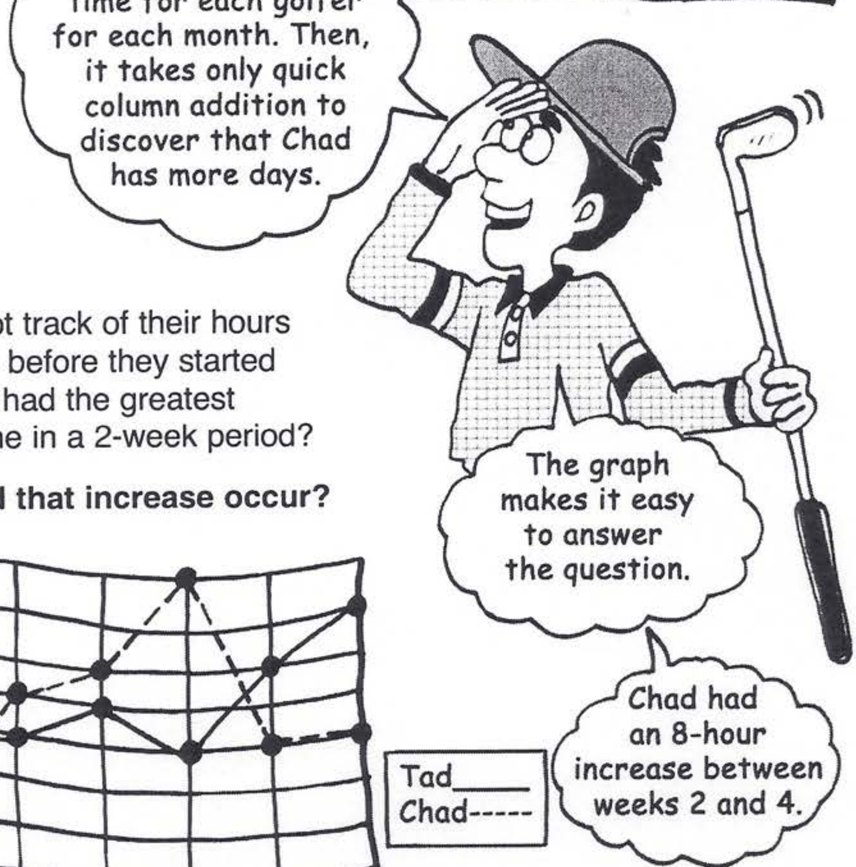
The Problem:

Two golfing friends traveled frequently to play in golf tournaments. In January, Chad traveled 14 days out of the month. Tad traveled $\frac{1}{2}$ the days that Chad traveled. In each of the next five months, Chad traveled two days more a month than in the previous month. In February through June, Tad traveled four days more each month than Chad had in February.

Month	Chad	Tad
Jan	14	7
Feb	16	20
Mar	18	20
Apr	20	20
May	22	20
Jun	24	20

Who traveled more days over the 6-month period?

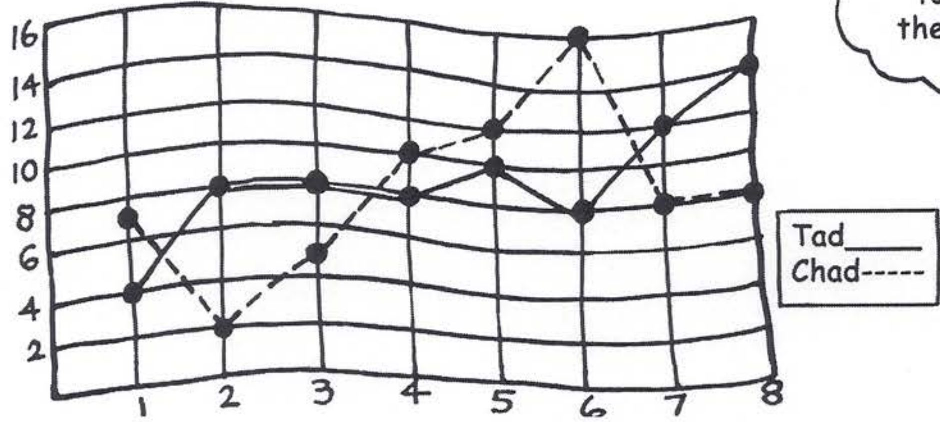




The Problem:

Chad and Tad also kept track of their hours of practice for 8 weeks before they started traveling. Which golfer had the greatest increase in practice time in a 2-week period?

Between which weeks did that increase occur?



Work Backwards

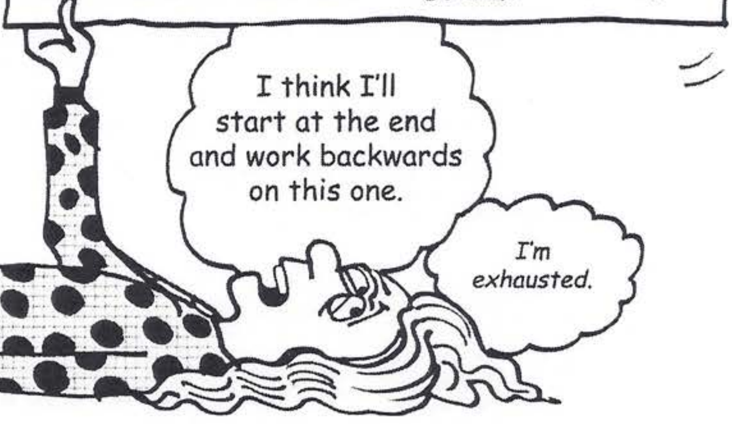
Sometimes it is helpful to start at the end of a problem and work backwards to find a missing fact. This is especially useful when a problem has a missing fact somewhere in the middle (or at the beginning) or many items of data. Here is one problem that calls for this approach.

The Problem:

Sabrina walked 30 minutes into town. After she got to town, she stopped at the shoe store where she spent 12 minutes choosing new athletic shoes and 21 minutes standing in line to pay for them. It took 4 minutes to get to the market, where she spent 7 minutes buying water and energy bars. She walked another 10 minutes to the gym, and did a workout that took 1 hour and 55 minutes. Then she jogged home in 21 minutes. She arrived home at 4:13 pm.

What time did she leave home?

arrived home		4:13
left gym	-21 min	3:52
arrived at gym	- 1 hr 55 min	1:57
left market for gym	- 10 min	1:47
arrived at market	-7 min	1:40
left shoe store	-4 min	1:36
got in line at shoe store	- 21 min	1:15
arrived at shoe store	- 12 min	1:03
left home for shoe store	-30 min	12:33



Use a Formula

Some problems just need a formula. Usually a formula is a shortcut to a solution, so be alert for chances to use one. Make sure you choose the correct formula and use it accurately.

The Problem:

That hungry Sabrina came home and ate a full box of veggie crackers.
The box measured 25 by 10 by 15 centimeters.
Each cubic centimeter in the box holds 0.1 grams of crackers.

What is the weight of the crackers she ate?

Use the formula for volume of a rectangular prism. Then, multiply the volume by 0.1 grams to calculate the weight.

$V = l \times w \times h$

$25 \times 10 \times 15 = 3750\ cm^3$

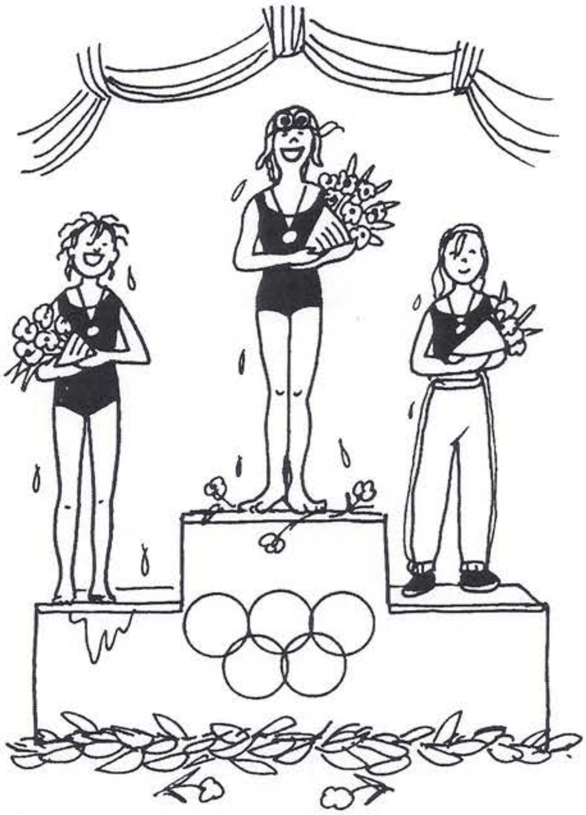
$3750 \times 0.1 = 375$ grams

THE FINAL COUNT

When the Olympic Games are over, the medals are counted. At the Summer Olympic Games in Atlanta, 842 medals were awarded. This is the way the final count looked for the top 20 medal-winning countries. Use the chart to solve the problems below.

Summer Olympic Games
Final Medal Standings for Top 20 Countries

Country	Gold	Silver	Bronze	Total Medals
United States	44	32	25	
Germany	20	18		65
Russia		21	16	63
China	16	22	12	
Australia	9		23	41
France		7	15	37
Italy	13	10	12	
South Korea	7	15		27
Cuba	9	8		25
Ukraine		2	12	23
Canada	3		8	22
Hungary		4	10	21
Romania	4		9	20
Netherlands	4	5		19
Poland	7	5	5	
Spain	5	6	6	
Bulgaria	3	7	5	
Brazil	3		9	15
Great Britain	1	8	6	
Belarus	1	6		15



1. Write the missing numbers in the spaces on the chart.
2. Which four countries won the same total number of medals? ________________ ________________
3. Which two countries won 17 medals? ________________
4. Which three countries each won 12 bronze medals? ________________ ________________ ________________
5. Which country won the same number of gold and bronze medals? ________________
6. Which country won 8 times as many bronze medals as gold medals? ________
7. Which country won twice as many medals as Cuba? ________________
8. Which country won more bronze medals than the U.S.? ________________
9. How many more gold medals did Russia win than Ukraine? ________________
10. Which country won half of its medals in silver? ________________

Use with page 233.

THE FINAL COUNT, CONT.

The medal count is quite different at the Winter Olympics because there are fewer events. The chart below tells the final medal count for all medal-winning countries at the Winter Olympic Games in Lillehammer, Norway. Use the information to solve the problems below.

1. Write the missing numbers in the spaces on the chart.

__________ 2. Find the total number of medals awarded in 1994.

__________ 3. How many medals did the top 6 countries win?

__________ 4. How many medals did the other 16 countries win?

__________ 5. Which country won more gold medals than Norway?

__________ 6. Which country won more silver and bronze, but fewer gold medals than the U.S.?

__________ 7. Which country won three times the gold medals of Canada?

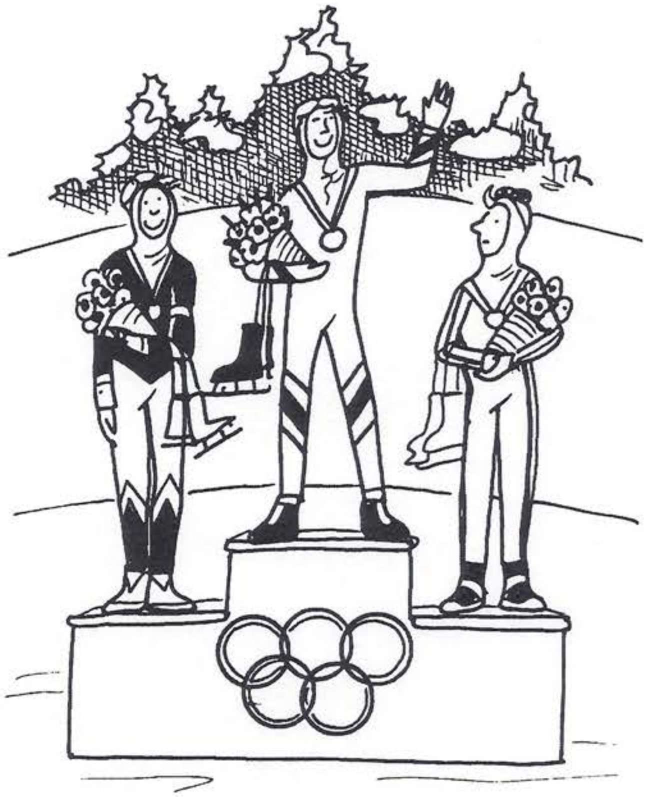
__________ 8. Which country won the same number of gold medals as Switzerland?

__________ 9. Which country won 17 more medals than China?

__________ 10. Which country won 22 fewer medals than Norway?

Winter Olympic Games
Final Medal Standings

Country	Gold	Silver	Bronze	Total Medals
Norway	10	11	5	
Germany	9	7		24
Russia	11		4	23
Italy	7	5	8	
United States	6		2	13
Canada		6	4	13
Switzerland		4	2	9
Austria	2	3	4	
South Korea		1	1	6
Finland	0		5	6
Japan	1	2		5
France	0		4	5
Netherlands	0	1	3	
Sweden		1	0	3
Kazakstan	1	2	0	
China	0		2	3
Slovenia	0	0		3
Ukraine		0	1	2
Belarus	0	2	0	
Great Britain	0		2	2
Uzbekistan	1	0		1
Australia		0	1	1



Use with page 232.

FLIPPING OVER NUMBERS

The Baker kids have been having a great time doing flips on their backyard trampoline. In order to keep track of everyone's total flips, Biff made a chart. By mistake, he left out some numbers. Finish the chart and answer the questions.

	Front Flips	Back Flips	Straddle Flips	Twist Flips	Totals
Biff	4		3	7	18
Bob	2	8	4	4	
Ben		1	6	10	22
BUD	3		3	3	12
Barb	7	9	1	1	
Bonnie	8	6		7	21
Totals					

1. Total Front Flips? _______
2. Who did the most flips? ________________
3. Three kids who tied? ________________ ________________ ________________
4. Who did the most Front Flips? ___________
5. Who did the least Straddle Flips? ___________
6. Who did the same number of all four kinds of flips? ________________
7. Who did the most Twist Flips? ___________
8. Who tied in Twist Flips? ________________
9. Kind of flip done most? ________________
10. Kind of flip done least? ________________
11. Barb's most successful flip? ______________
12. Bonnie's least successful flip? ___________
13. Whose total was 10 more than Bud's? ______________
14. What flip total was $\frac{1}{3}$ of Bonnie's total flips? ______________
15. Who flipped 3 more than Barb? ___________
16. Total of all flips? _______

OUTRAGEOUS COLLECTIONS

What strange things some people collect! Many people have collections, but some take it to extremes. People who collect thousands of magnets, clovers, mousetraps, or airsickness bags may do this for the love of collecting. Or they may do it to get in the record books!

The table below shows **data** (numerical information) for some outrageous record collections. Use the information on the table to answer the questions.

1. Which collection is the largest? ______
2. Which collection is the smallest? ______
3. Who has more items, Hugh or Ted? ______

4. How many clovers are in George's collection? ______
5. Who collected four times as many items as Sonja? ______
6. Who collected about five times as many items as Louise? ______
7. Which 3 collections are very close to 2000? ______

OUTRAGEOUS COLLECTIONS

Collection	Collector	Record Number Collected
mousetraps	Reinhard Hellwig	2,334
golf balls	Ted J. Hoz	43,824
items of underwear	Imelda Marcos	1,700
shoes	Sonja Bata	10,000
watches	Florenzo Barindelli	3,562
light bulbs	Hugh Hicks	60,000
airsickness bags	Nick Vermeulen	2,112
gnomes & pixies	Anne Atkin	2,010
bandages (unused)	Brian Viner	3,750
refrigerator magnets	Louise J. Greenfarb	21,500
clovers	George Kaminski	13,382 four-leaf 1,336 five-leaf 78 six-leaf 6 seven-leaf
bubble gum	Thomas & Volker Martins	1,712
parking meters	Lotta Sjölin	292
nutcrackers	Jürgen Löschner	2,200
ballpoint pens	Angelika Unverhau	108,500
piggy banks	Ove Nortstrom	3,575
jet fighters	Michel Pont	100
marbles	Sam McCarthy-Fox	40,000

8. Which collection has about the same number as the bubble gum collection? ______
9. How many collections are larger than the underwear collection? ______
10. How many collections are smaller than the piggy bank collection? ______
11. Whose collection is about the same in number of items as the watches? ______
12. Which collection surprises you most? ______
    Why? ______

Use with page 277.

OUTRAGEOUS COLLECTIONS, CONT.

Curious visitors come to see many of the world record-setting collections described on page 276. This table is about visitors to some other collections that have not set any records. Use the table to answer the questions.

CURIOUS VISITORS

Numbers of Visitors May-June

Collection Visited	May	June	July	August
Elvis souvenirs	**666**	**900**	**1,001**	**768**
ski poles	**89**	**320**	**465**	**345**
shoes	**1,000**	**1,590**	**1,899**	**1,200**
marbles	**707**	**933**	**955**	**700**
safety pins	**30**	**66**	**71**	**14**
stuffed animals	**4,500**	**4,811**	**5,736**	**4,801**
spoons	**691**	**699**	**741**	**366**
gum wrappers	**190**	**580**	**711**	**533**
cash registers	**1,101**	**2,801**	**4,138**	**3,100**
lightbulbs	**1,400**	**1,451**	**1,478**	**1,410**
shoelaces	**57**	**84**	**99**	**150**

1. For which collection was the number of visitors about the same over all four months? ____________
2. Which month was the best for most of the collections? ____________
3. Most collections had (more, less) visitors in June than July. ____________
4. Which collection had fewer visitors in July than in August? ____________
5. Most collections had (more, less) visitors in August than in July. ____________
6. How many collections had fewer visitors in August than in May? ____________
7. Which collection had the least interest from visitors over the summer? ____________
8. Which collection seems to have had the most visitors over the summer? ____________
9. How many collections had more visitors in July than the shoe collection did? ____________
10. How many collections had less visitors in May than the marble collection did? ____________
11. Which collection had about 2,000 more visitors in August than it did in May? ____________
12. How many more visitors saw the safety pin exhibit in June than in August? ____________

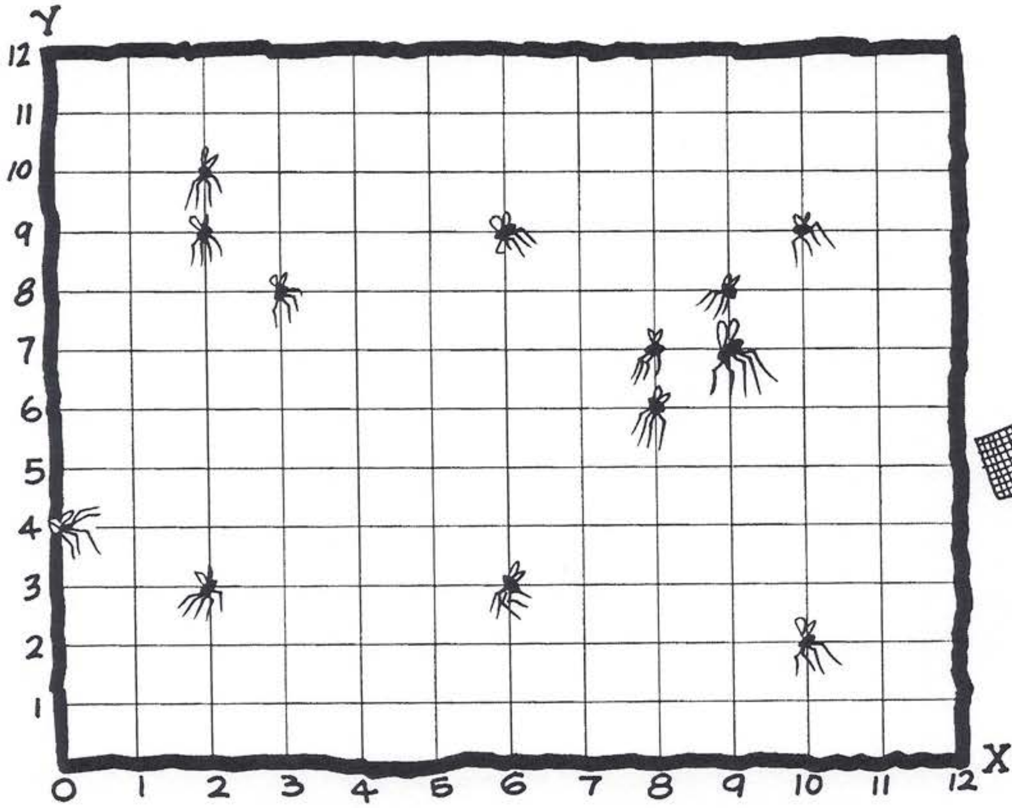
Use with page 276.

Find Locations on a Coordinate Grid

SWATTING TO SET A RECORD

Yes, there really is a World Championship Mosquito-Killing Championship. It is held every year in Finland. Henri Pellonpää holds the record for the most of the pesky insects killed in five minutes. His record is 21 mosquitoes.

Follow the directions below to locate Henri's mosquitoes on the coordinate grid.



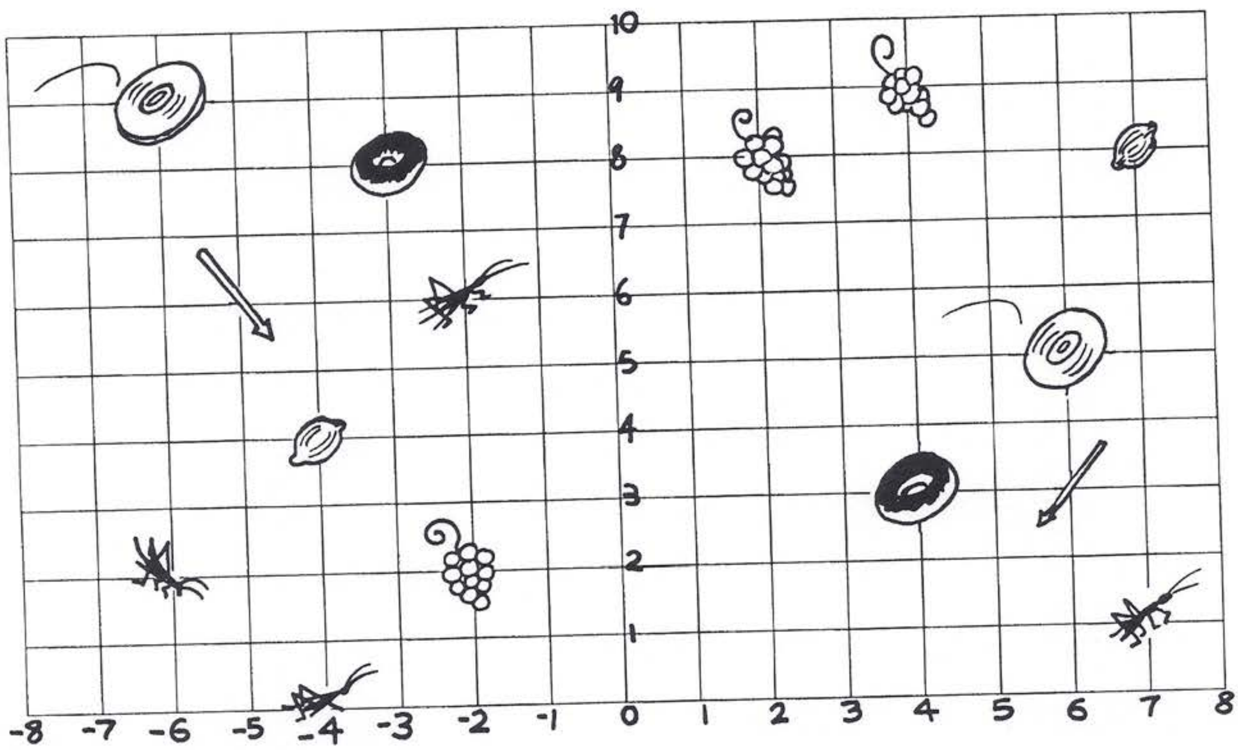
Remember that coordinates of a location are written (x, y). X is the location on the horizontal line. Y is the location on the vertical line.

1. Is there a mosquito at (8, 6)? ________
2. Is there a mosquito at (2, 10)? ________
3. Is there a mosquito at (9, 4)? ________
4. Is there a mosquito at (10, 8)? ________
5. Is there a mosquito at (12, 0)? ________
6. Is there a mosquito at (9, 7)? ________
7. Is there a mosquito at (8, 7)? ________
8. Is there a mosquito at (3, 6)? ________
9. Is there a mosquito at (6, 3)? ________
10. Where is the largest mosquito? ________
11. Draw a mosquito at (12, 3).
12. Draw a mosquito at (3, 6).
13. Draw a mosquito at (9, 4).
14. Draw a mosquito at (8, 0).

THE THINGS PEOPLE THROW

It's amazing what things get thrown, tossed, and spit in an effort to set a record! Some of the world records include spitting cherry pits and crickets, tossing pancakes and cow pies, and catching tossed grapes in the mouth.

Find the tossed items on the coordinate grid below. Answer the questions about their locations.



Write the locations where these are found. Write the coordinates like this: (x, y).

1. The  are found at __________ , __________ , and __________ .

2. The  are found at __________ and __________ .

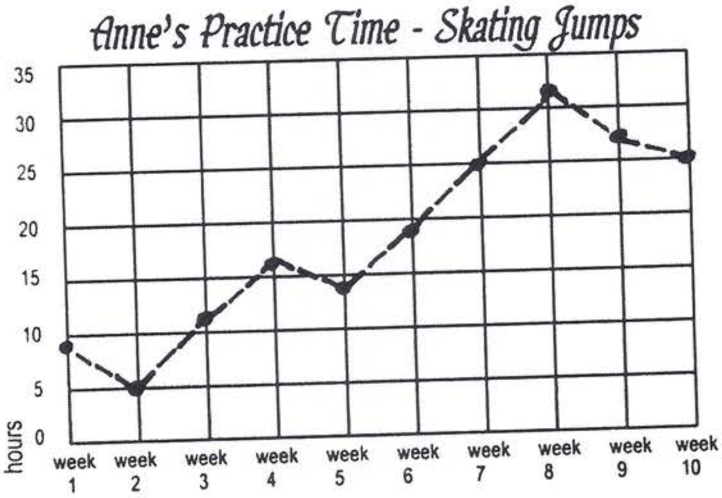
3. The  are found at __________ and __________ .

4. The  are found at __________ and __________ .

5. The  are found at __________ and __________ .

6. The  are found at __________ , __________ , __________ , and __________ .

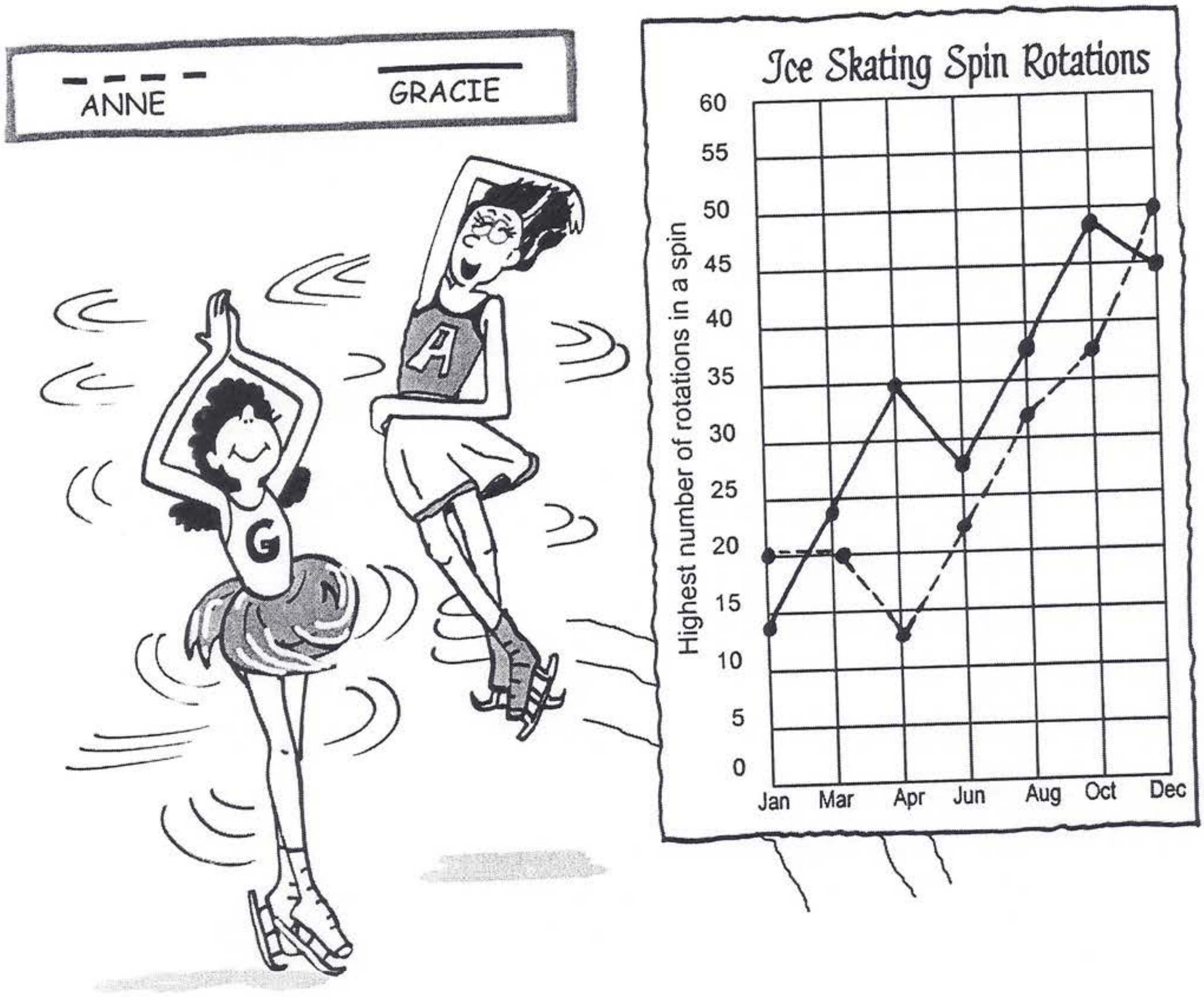
Line Graphs



A **line graph** uses a line on a grid to show data over time. A line graph has a special talent that other kinds of graphs don't have: it is able to show **changes** in data over a period of time.

This graph shows the number of hours Anne practiced her ice skating jumps over a period of ten weeks.

The graph below uses two different lines to compare the data on spins for two skaters. The line shows changes in number of rotations in their spins over a 12-month period.



Tree Diagrams

A **tree diagram** is an interesting and helpful visual tool for figuring probability. All the possible outcomes for independent events can be shown on a tree diagram.

> **Get Sharp Tip #29**
> A tree diagram is a good tool to use to show outcomes for two independent events.

Angie got to lunch late.

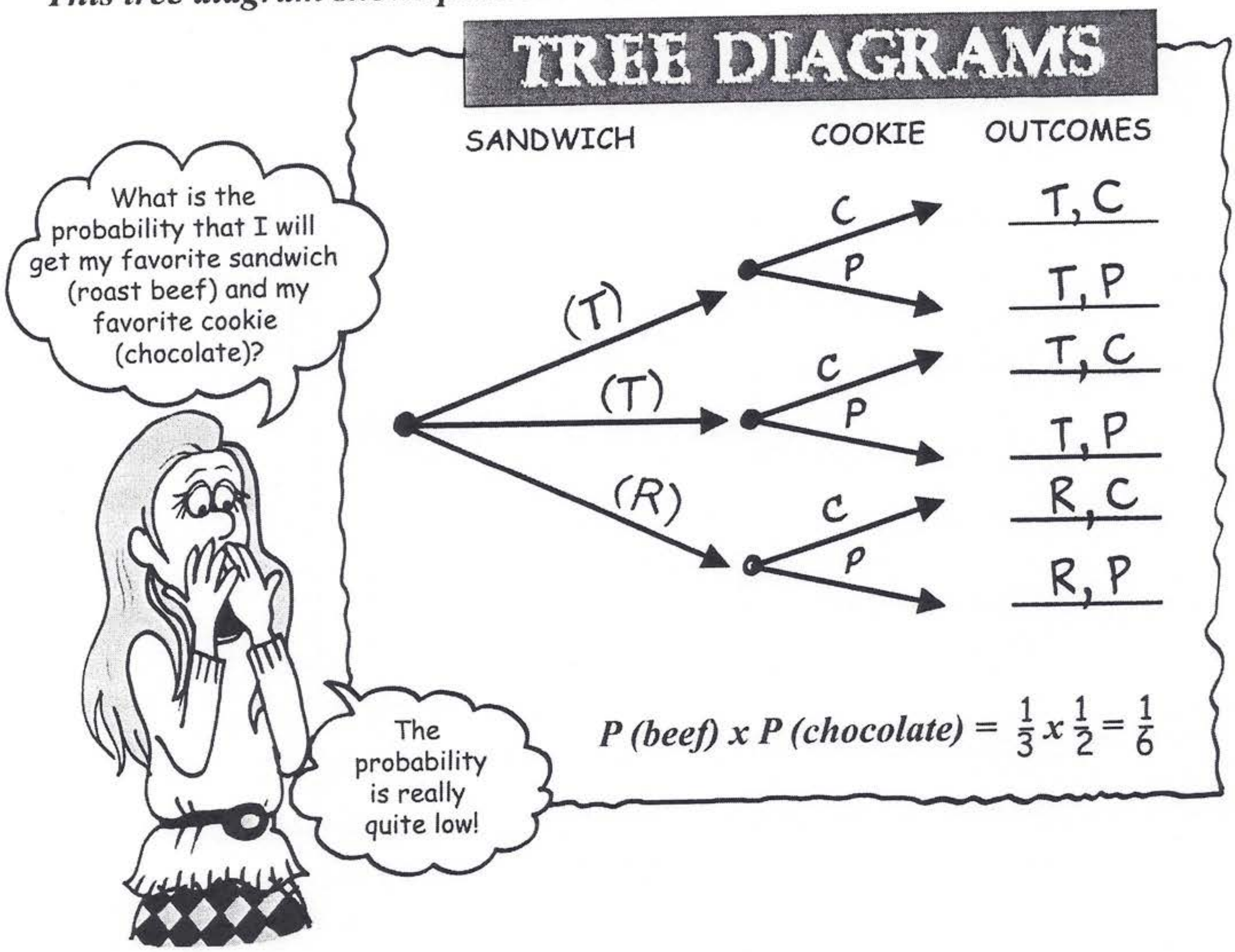
There were 3 sandwiches left: *2 turkey and 1 roast beef.*
There were 2 cookies left: *1 chocolate and 1 peanut butter.*

All cookies and sandwiches were wrapped, but had no labels. Angie took one sandwich and one cookie.

The tree diagram shows the possible outcomes for Angie's choices.

T = turkey sandwich	*C = chocolate cookie*
R = roast beef sandwich	*P = peanut butter cookie*

This tree diagram shows possible outcomes:

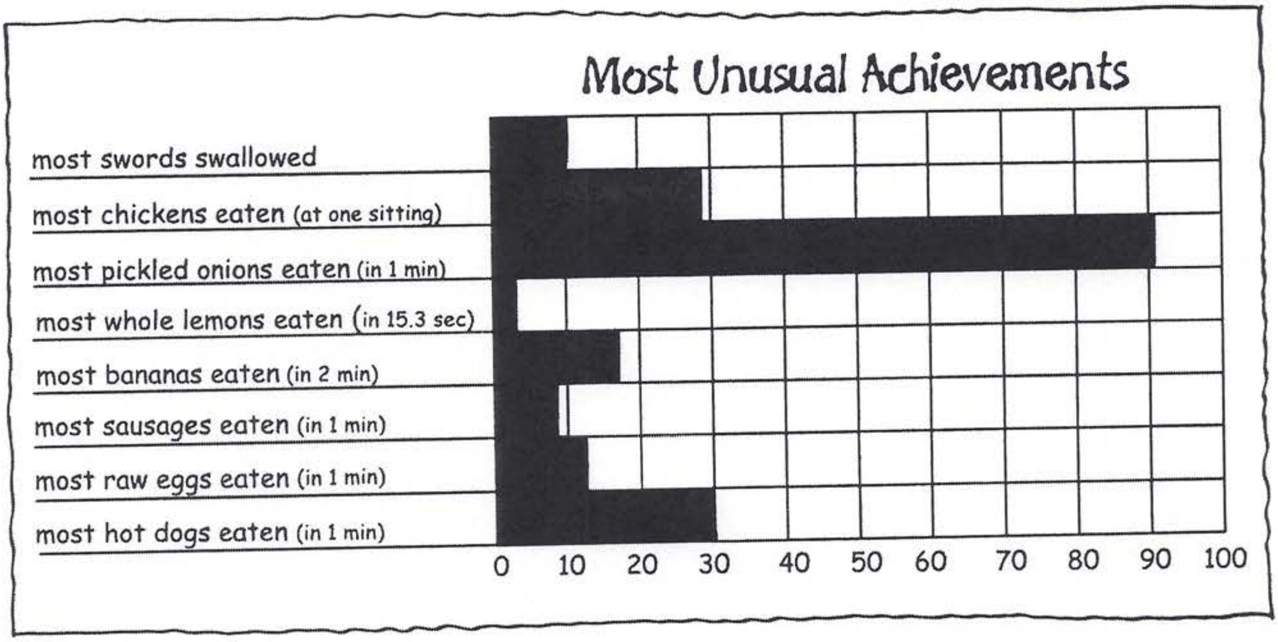


$$P\ (beef)\ x\ P\ (chocolate) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

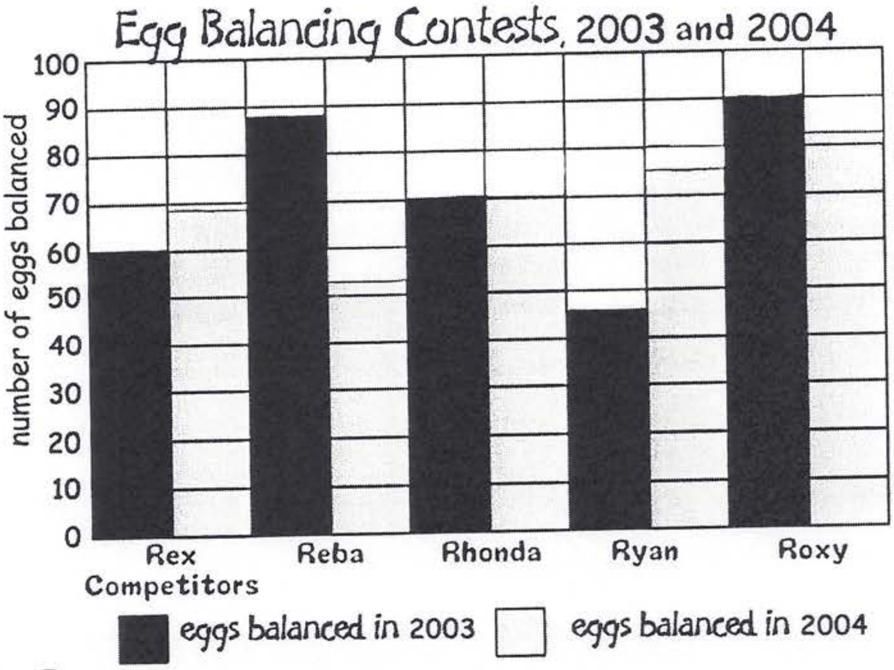
Bar Graphs

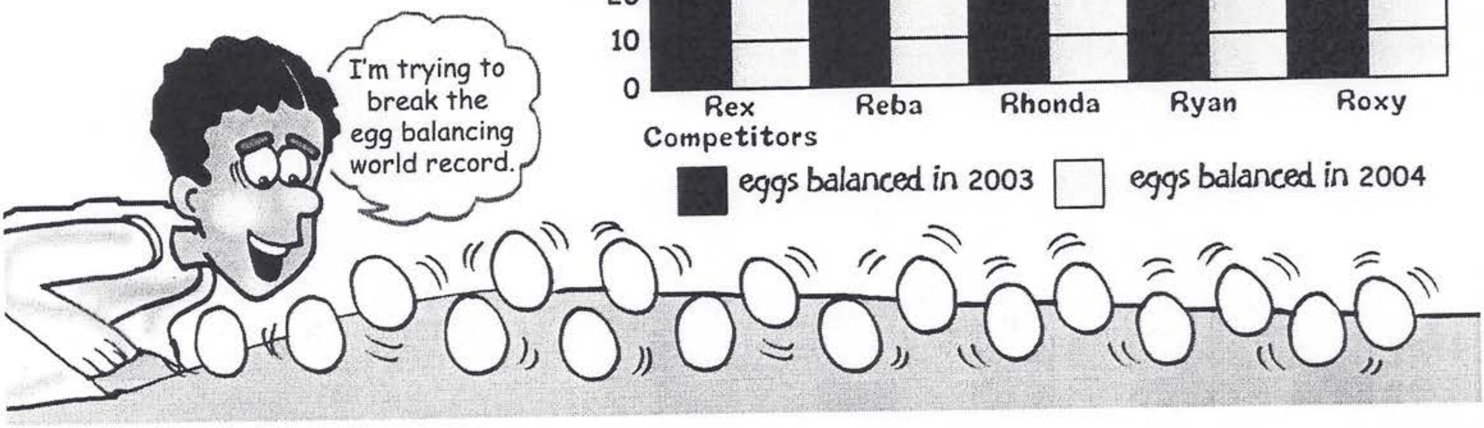
A **bar graph** uses bars of different lengths to show and compare data.

A **single bar graph** shows one kind of data. This single bar graph shows the number of things accomplished in different kinds of contests.



A **double bar graph** shows two kinds of data at once. This graph uses two colors of bars to show the number of eggs each person balanced in each of two different egg-balancing competitions.





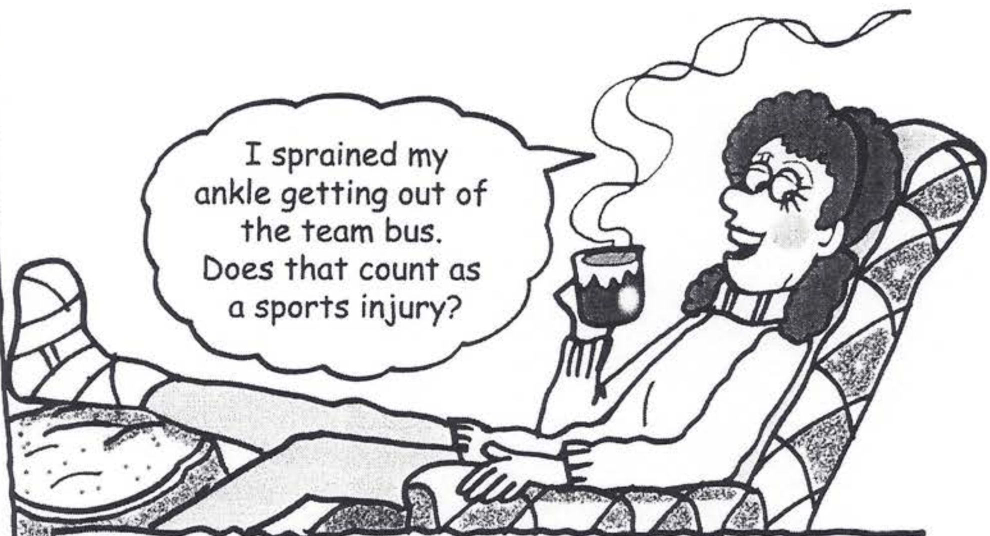
Histograms

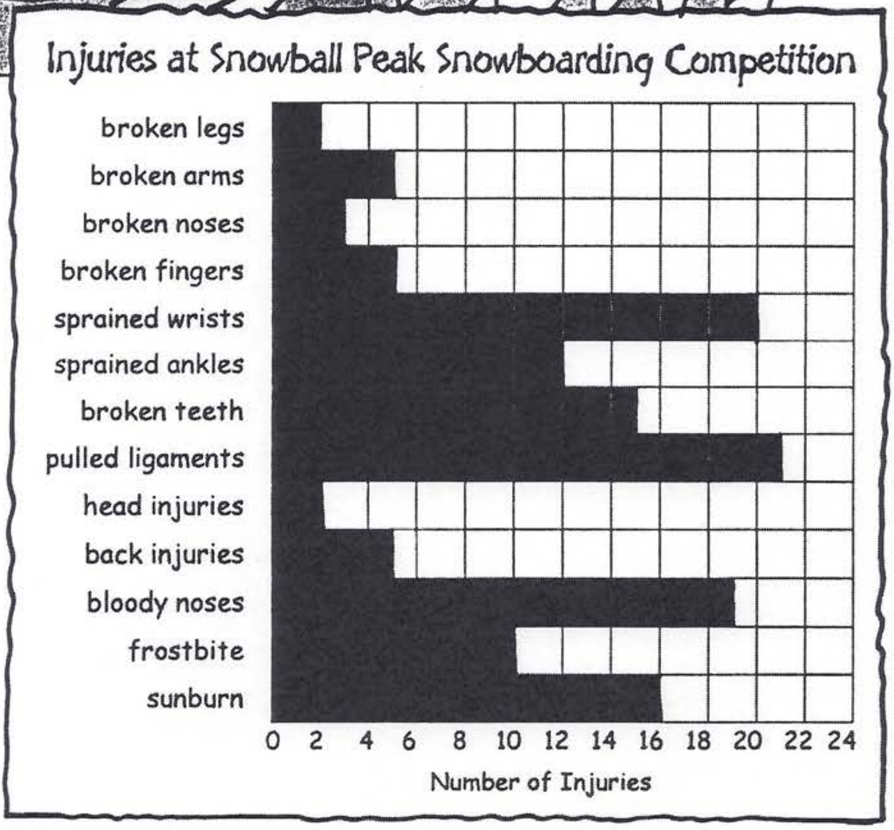
A **histogram** is a bar graph that shows frequency data (or, how often something happens). This data is about the number of injuries at a week-long snowboard competition. Data on the different kinds of injuries has been collected and organized on the table.

This same data is displayed below in graph form. From reading the graph, you can see the frequency with which each type of injury occurred.

INJURIES
Snowball Peak Competition

Kind of Injury	Frequency
broken legs	2
broken arms	5
broken noses	3
broken fingers	5
sprained wrists	20
sprained ankles	12
broken teeth	15
pulled ligaments	21
head injuries	2
back injuries	5
bloody noses	19
frostbite	10
sunburn	16





After watching the 2002 Olympic snowboard competitions, 18.6 million people said they would like to try the sport.

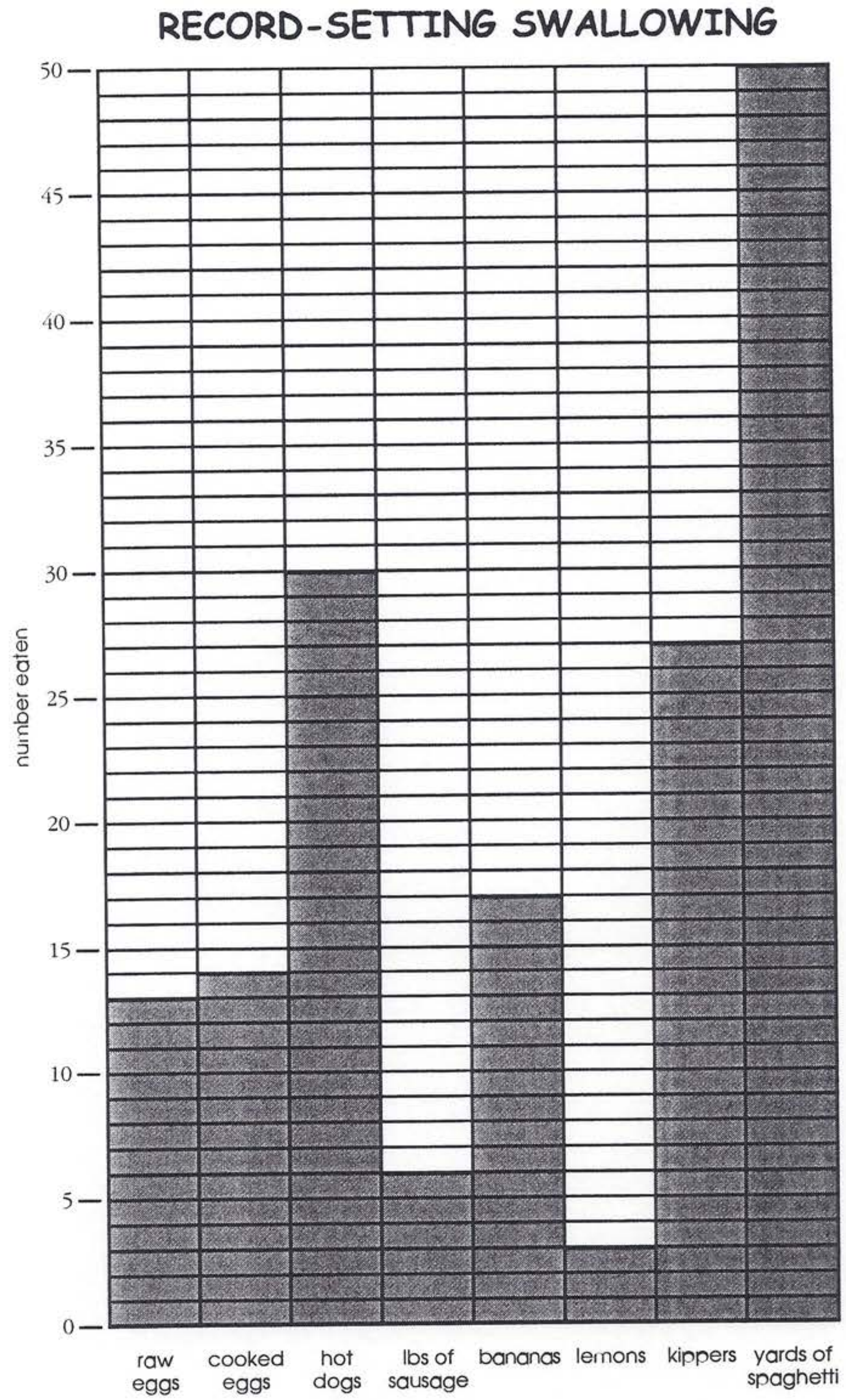
15,000 spectators watched Kelly Clark win the women's snowboarding halfpipe competition at the 2002 Winter Olympics.

Read & Interpret a Bar Graph
Use Information on Graph to Solve Problems

RECORD-SETTING SWALLOWING

Large amounts of food items are swallowed in extremely fast times to set food-eating records. Pancakes, spaghetti, raw eggs, whole lemons, pickled onions and other interesting foods are gobbled up for the sake of competition.

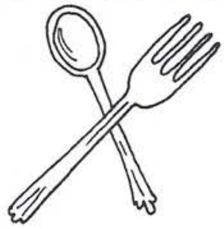
The graph shows the number of food items that were eaten to set some speed-eating records. Use the graph to solve the problems on the next page.



Use with page 283.

Read & Interpret a Bar Graph
Use Information on Graph to Solve Problems

RECORD-SETTING SWALLOWING, CONTINUED



1. How many eggs were eaten to set the two egg records?

____

2. How many less bananas than kippers were eaten to set the record?

____

3. The raw eggs were eaten in 1 second. The cooked eggs took 14.42 seconds. How much longer did the cooked eggs take?

____

4. How many more bananas were eaten for the banana record than lemons were eaten for the lemon-eating record?

____

5. It took 2 minutes to set the banana record. At this rate, how many bananas could be eaten in 10 minutes?

____

6. It took 3 minutes and 10 seconds to eat the amount of sausage shown on the graph. At the same rate, how long would it take to eat 30 pounds?

____

7. It took 64 seconds to eat the hot dogs for the record. About how much time did it take per hot dog?

____

8. It took one second to set the raw egg record. At this rate, how many raw eggs could be eaten in a minute?

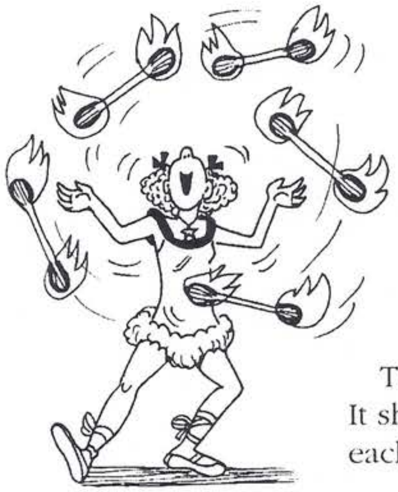
____

9. The actual spaghetti-eating record was set by eating 100 yards of spaghetti. If the amount on the chart took 6.01 seconds, how long did the actual record take?

____

10. The lemons were eaten whole—skins, seeds, and all—in 15.3 seconds. At the same rate, how long would it take to eat 9 lemons? _______________

Use with page 282.

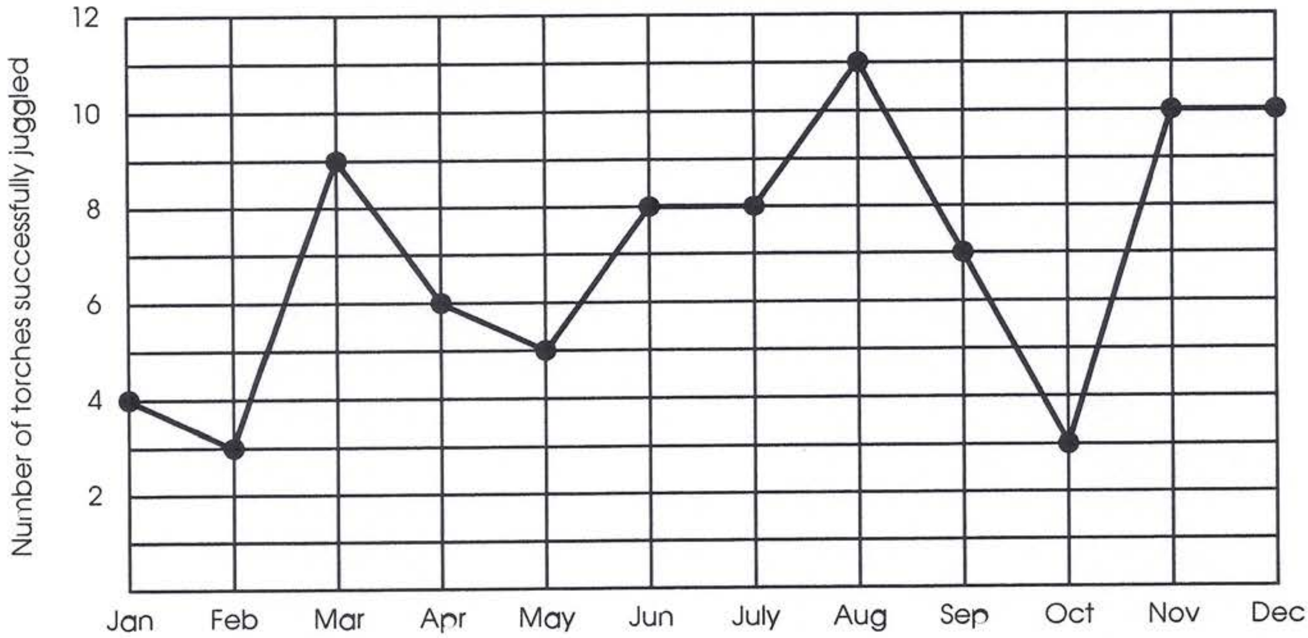


PLAYING WITH FIRE

Anthony Gatto, of the United States, set the record for flaming torch juggling by keeping seven flaming torches moving in the air at once. This is a record you should not try practicing in your home or back yard!

The line graph shows the results of one juggler's practice for a year. It shows the most torches she successfully juggled at any time during each month. Use the graph to find information about Jasmine's juggles.

JUGGLING FLAMING TORCHES



Write . . .

1. Number juggled in March ______________
2. Number juggled in November ______________
3. Number juggled in May ______________
4. Best month ______________
5. Worst month ______________
6. Difference between August and September ______________
7. Difference between November and December ______________
8. Greatest drop between which 2 months?

   ____

   ____
9. Greatest increase between which 2 months?

   ____

   ____
10. Difference between least and greatest numbers?

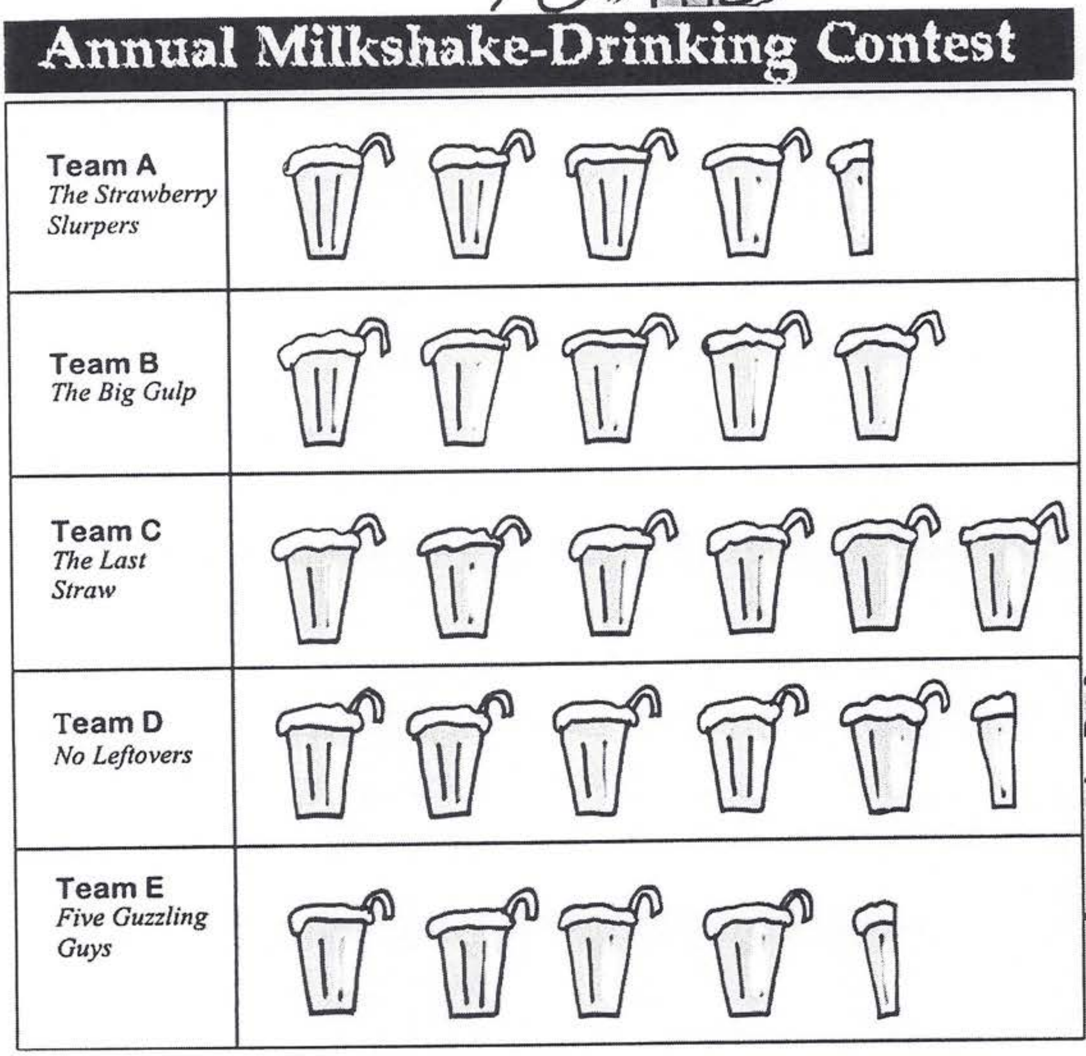
    ____
11. Difference between March and April?

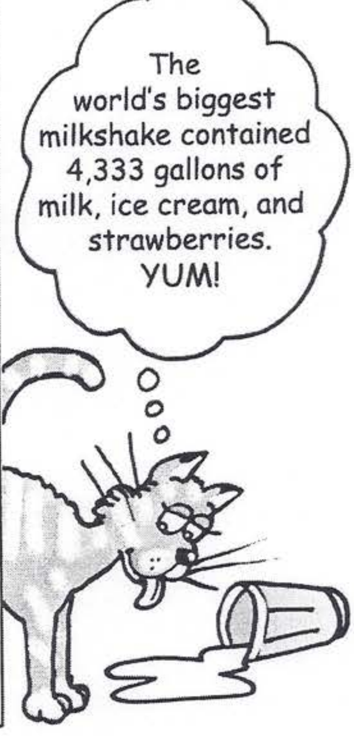
    ____
12. Difference between January and December?

    ____

Pictographs

A **pictograph** uses pictures, symbols, or icons to display and compare data. Each picture represents a specific data number. Fractions of the picture are used to represent fractions of the data amount. A key shows what the picture represents.



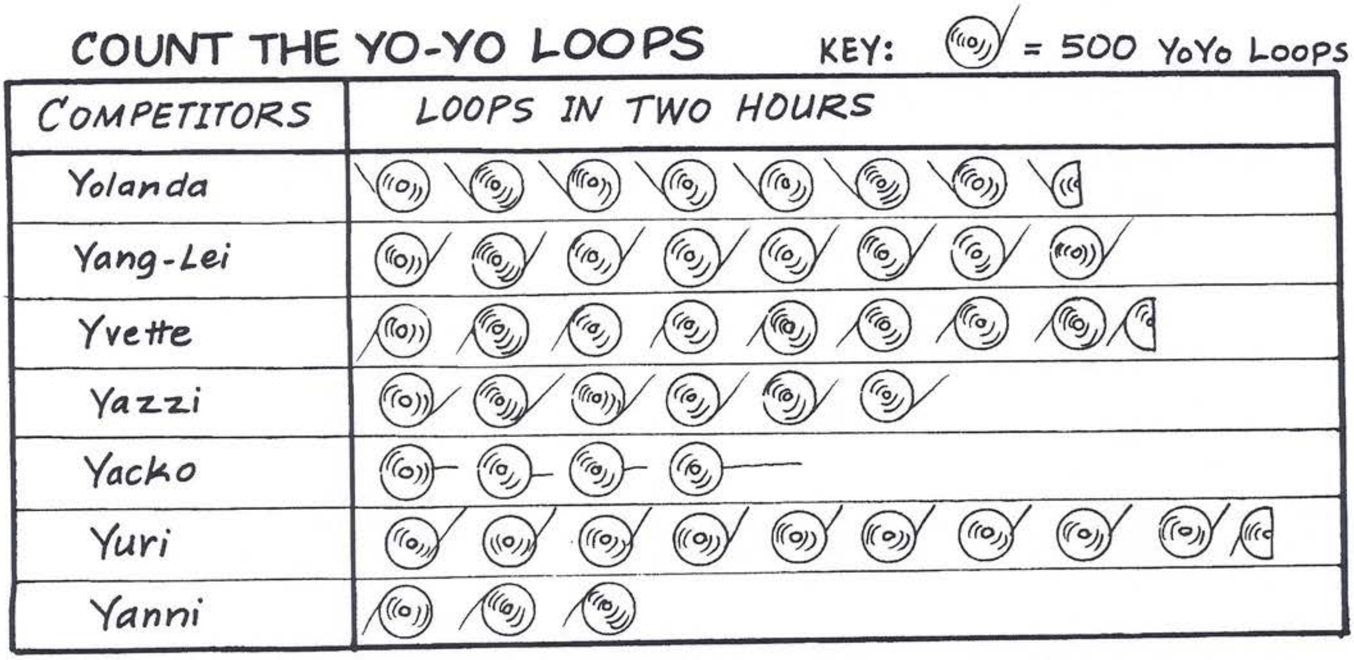


Read & Interpret a Pictograph

LOTS & LOTS OF LOOPS

Eddy McDonald of Canada spent three hours performing loops with his yo-yo. Someone was counting as he was doing loops, and they counted 21,663 complete loops. He must have had a tired arm!

The **pictograph** uses pictures to show numbers of yo-yo loops done by some less expert yo-yo spinners. Use the graph to tell whether each statement is true or false.



Write T or F for each statement.

______ 1. Yacko did 4500 loops.

______ 2. All seven did over 30,000 loops.

______ 3. Yanni did half the loops of Yazzi.

______ 4. Yang-Lei did twice the loops as Yacko.

______ 5. Yolanda did 1000 less loops than Yvette.

______ 6. Yolana and Yazzi together did 6750 loops.

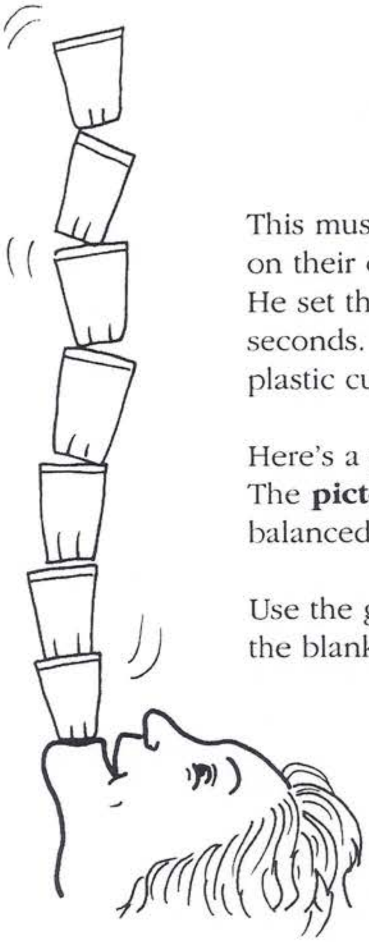
______ 7. Yacko and Yvette together did 7000 loops.

______ 8. Yvette did three times as many loops as Yanni.

______ 9. Yuri did more than three times as many as Yanni.

______ 10. All seven put together did less than Eddy McDonald when he set his record.

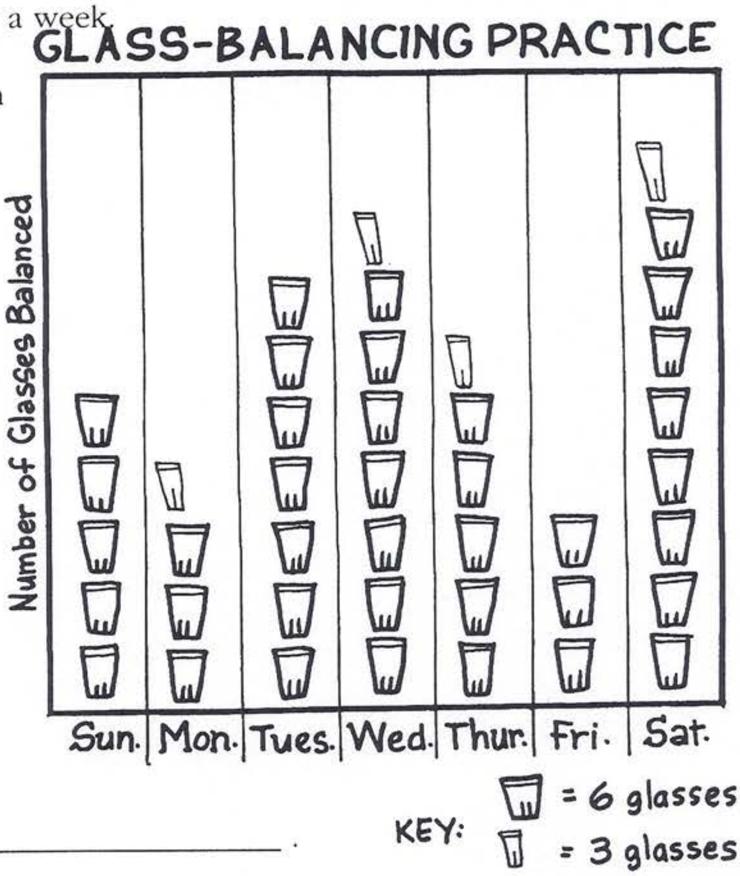
BODACIOUS BALANCING



This must take years of practice. People actually balance stacks of glasses on their chins! Ashrita Furman is the record-holder for this amazing trick. He set the record by balancing a stack of 62 glasses on his chin for ten seconds. DON'T try this at home with real glasses! Only practice with plastic cups!

Here's a graph that another glass-balancer kept as she practiced her skill. The **pictograph** uses pictures to show the amounts of glasses successfully balanced each day for a week.

Use the graph to fill in the blanks below.



1. On Friday, she balanced _____ glasses.
2. On Thursday, she balanced _____ glasses.
3. On Tuesday, she balanced _____ glasses.
4. The best day for balancing was ____________________ .
5. The worst practice day was ____________________ .
6. She balanced 12 more on Tuesday than on ____________________ .
7. She balanced 18 less on Thursday than on ____________________ .
8. Gladys balanced fewer glasses on ____________________ than on Monday.
9. _____ glasses were successfully balanced on Wednesday.
10. The best 3 days in a row for practice were ____________________ through ____________________ .
11. The average number of glasses balanced over the week was _____ .
12. Her best practice was _____ less glasses than Ashrita Furman's record.

Frequency

Get Sharp Tip #27

Relative frequency is:

$$\frac{\text{frequency of an item}}{\text{total of frequencies}}$$

The relative frequency of riders over 75 years old is 2:50 or $\frac{2}{50}$.

The **frequency** of a number means how often it appears in a set of data. Sometimes it is useful or necessary to find out how often a number (or group of numbers) shows up in a set of data.

This set of data shows the ages of all the competitors who registered for a bullriding event in a rodeo. As each rider registered, his or her age was written on a list.

To find out how many of the riders fell into certain age groups, the data was organized into a frequency table.

1. First, the ages were grouped into intervals (15–24, 25–34, and so on).
2. Next, a tally mark was placed into the correct tally column each time an age was recorded.
3. Finally, the tally marks were counted.
4. The frequency of ages in each group was written as a number in the ***frequency*** column.

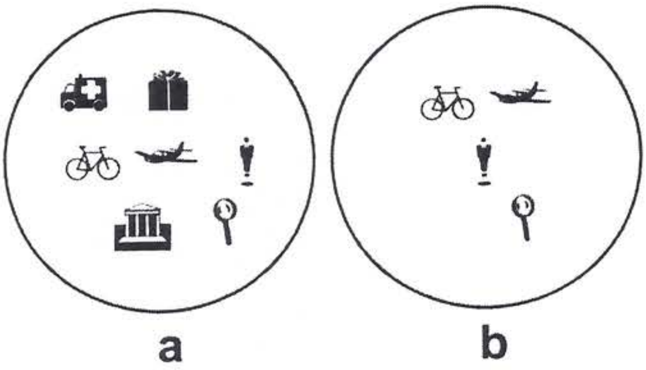
Ages of Bull-Riding Competitors

22	26	16	56	40	36
68	20	17	20	41	47
15	18	23	18	26	19
32	46	29	60	17	56
51	66	35	28	29	24
40	76	49	33	68	34
30	77	50	66	31	30
16	16	20	26	31	23
20	40				

Ages of Bull-Riding Competitors

Ages	Tally	Frequency
15-24	𝍸 𝍸 𝍸 ||	17
25-39	𝍸 𝍸 𝍸	15
40-54	𝍸 ||||	9
55-64	|||	3
65-75	||||	4
over 75	||	2

Subsets are sets made of any member of a set or any combination of members of a set.

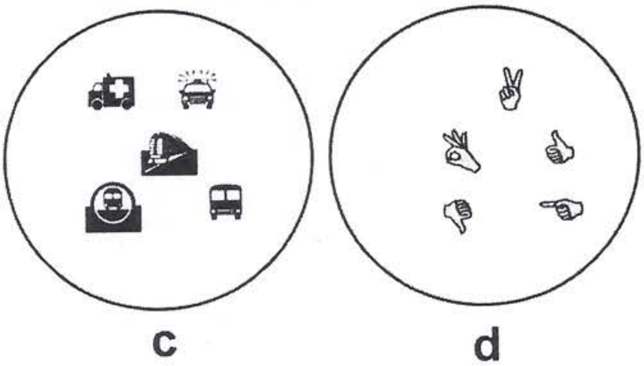


B is a subset of **A**.

The symbol $\subset$ means ***is a proper subset of***.

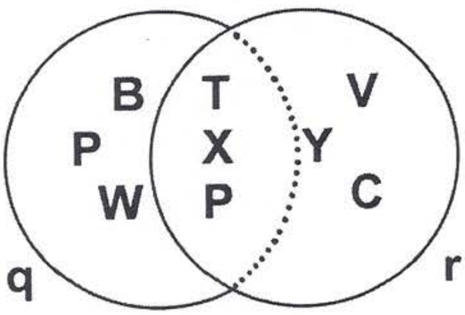
B $\subset$ **A**.

Equivalent sets are sets having the same number of members.



C and **D** are equivalent sets.

An **intersection of sets** is the set of members common to each of two or more sets.

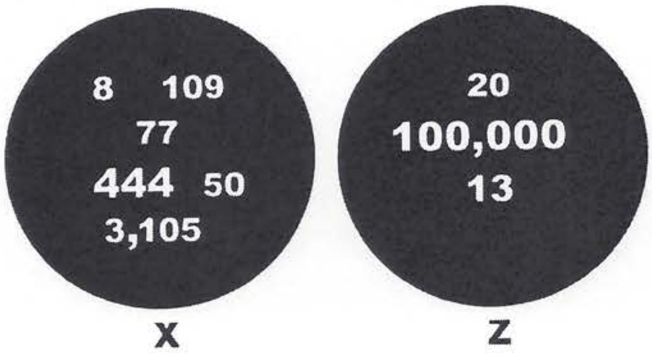


The symbol $\cap$ represents the intersection of sets.

Q $\cap$ **R**.

The intersection of sets **Q** and **R** is T, X, and P.

A **union of sets** is a set containing the combined members of two or more sets.



The symbol $\cup$ represents the union of sets.

X $\cup$ **Z.**

The union of sets **X** and **Z** is $\{8; 13; 20; 50; 77; 109; 444; 3{,}105; 100{,}000\}$.

Statistics

Statistics is a branch of mathematics that deals with numerical information (called **data**).
Statistics involves collecting, organizing, interpreting, presenting, and using data.

Collecting data allows you to compare data.

One good way to show data is to organize it into a table.

Get Sharp Tip #25

Make sure any table of data you create has a clear title and clear labels for all rows and columns.

Ski-Jumping Distances
(Sky High County Ski Club)

Jumper	First Jump (distance in feet)	Second Jump (distance in feet)
Sam	568	500
Pam	310	308
Stan	610	478
Dan	540	603
Fran	344	319
Jan	311	298
Van	462	591
Graham	603	624
Nan	285	333

CURIOUS RECORDS
(Distances rounded to the nearest kilometer)

Event	Record Distances
run holding a raw egg on a spoon	42 km
sailing in a bathtub	146 km
pushing a baby carriage (in 24 hours)	437 km
pushing a bathtub (in 24 hours)	514 km
racing downhill on skis (in one race)	15.8 km
crawling	1,399 km
travel in a lifeboat	1,287 km
travel in a wheelchair	40,076 km
travel on a lawnmower	23,488.5 km

Range, Mean, Median, & Mode

To understand statistics, you need to know about **range, median, mode *and* mean.** These are some of the most important words in statistics, because they help to describe sets of data.

Number of Minor Injuries During Ski Jumping Competition

Competitor's Name	A.J. Snow	J.R. Crash	Todd Rayce	Gabe McTrick	Abby deWheel	Z.Z. Tubes	Flip Slykes	D.D. Wynn	C. C. Cross
Number of scrapes, cuts, bruises, sprains	10	6	16	14	20	6	13	8	6

Range is the difference between the least and the greatest numbers in the set of data.

The range here is 6 – 20 injuries.

Get Sharp Tip #26
To find the median, first arrange all the data items into numerical order.

Mean is the average of the data. To find the mean, divide the sum of all the data by the number of items.

For this data, the mean =

$$\frac{10+6+16+14+20+6+13+8+6}{9} = \frac{99}{9}$$

The mean is 11.

Median is the number in the middle of a set of data.

The numbers in this set are:
6 – 6 – 6 – 8 – 10 – 13 – 14 – 16 – 20

The median is 10.

Mode is the number that appears most often in a set of data.

The mode is 6.

Sometimes there is no mode.
Sometimes there are two or more modes.